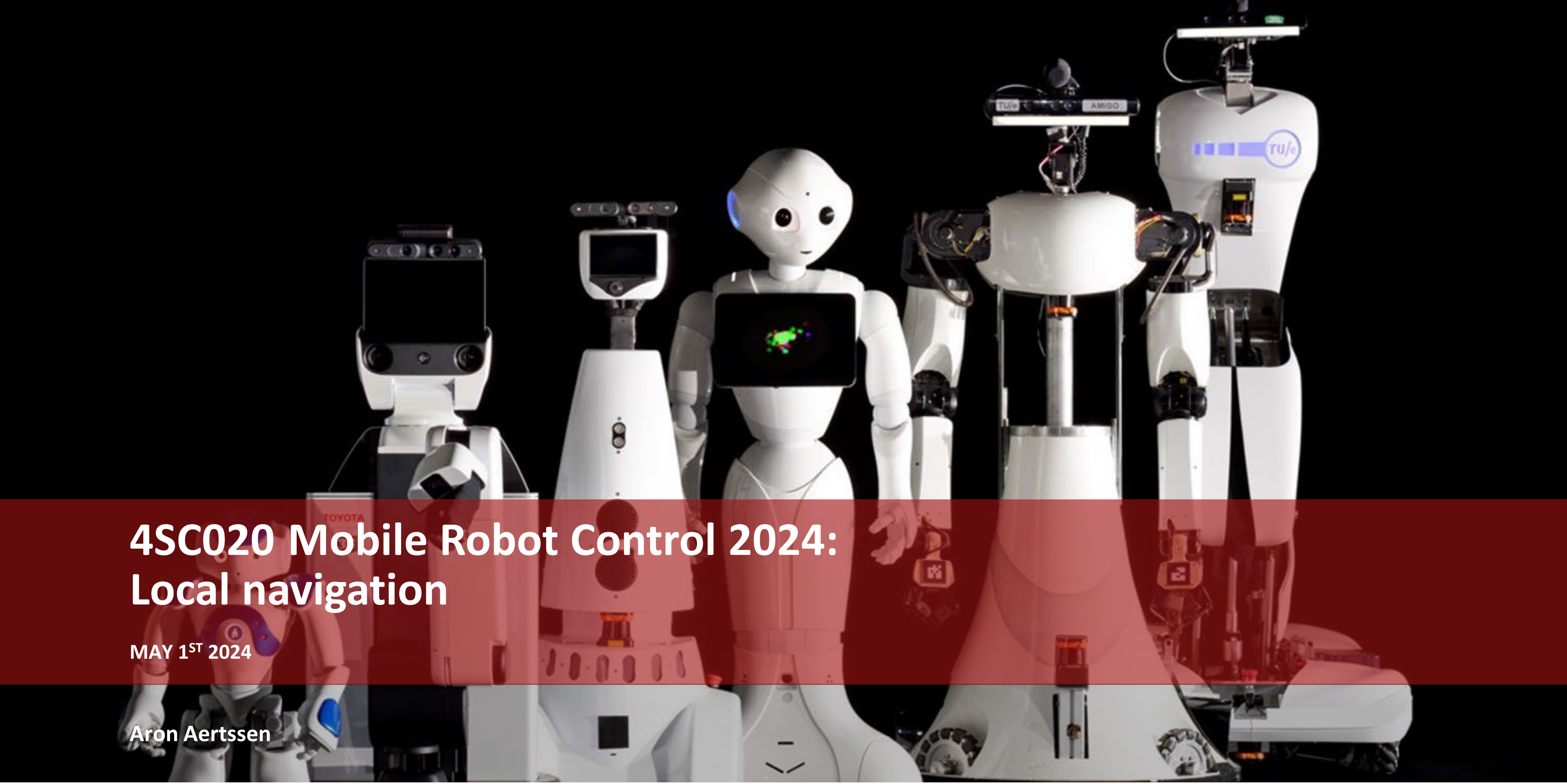


4SC020 Mobile Robot Control 2024: Local navigation

MAY 1ST 2024

Aron Aertssen

Mechanical Engineering, Robotics

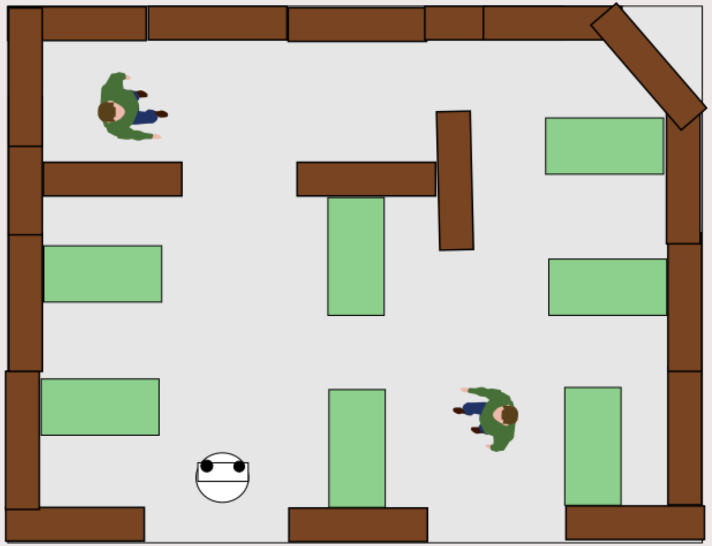
EINDHOVEN UNIVERSITY OF TECHNOLOGY

Outline

- Robot navigation problem
- Local navigation algorithms: properties
- Local navigation algorithms: examples
- Recap
- Assignment

Robot navigation problem / introduction

- What is the robot navigation problem?



Robot navigation problem / introduction

- What is the robot navigation problem?
  - Find a feasible path or trajectory from a given initial pose (A) to the desired final pose (B)

Robot navigation problem / introduction

- What is the robot navigation problem?
  - Find a feasible path or trajectory from a given initial pose (A) to the desired final pose (B)
- This raises a question: where is A and where is B?

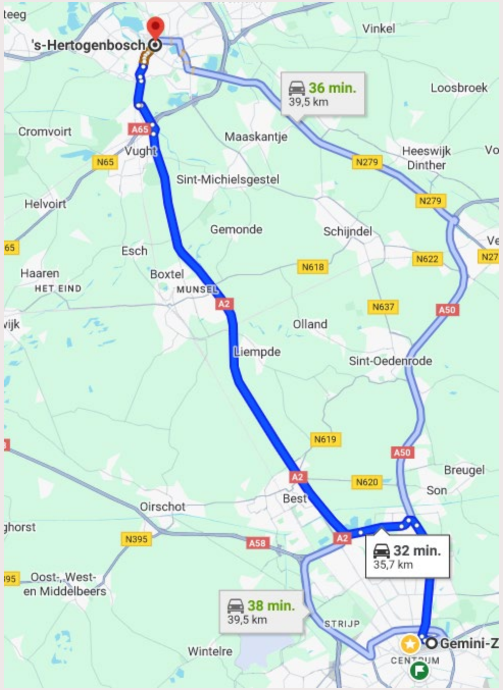
Robot navigation problem / local vs. global

- Division into global and local navigation. Why?

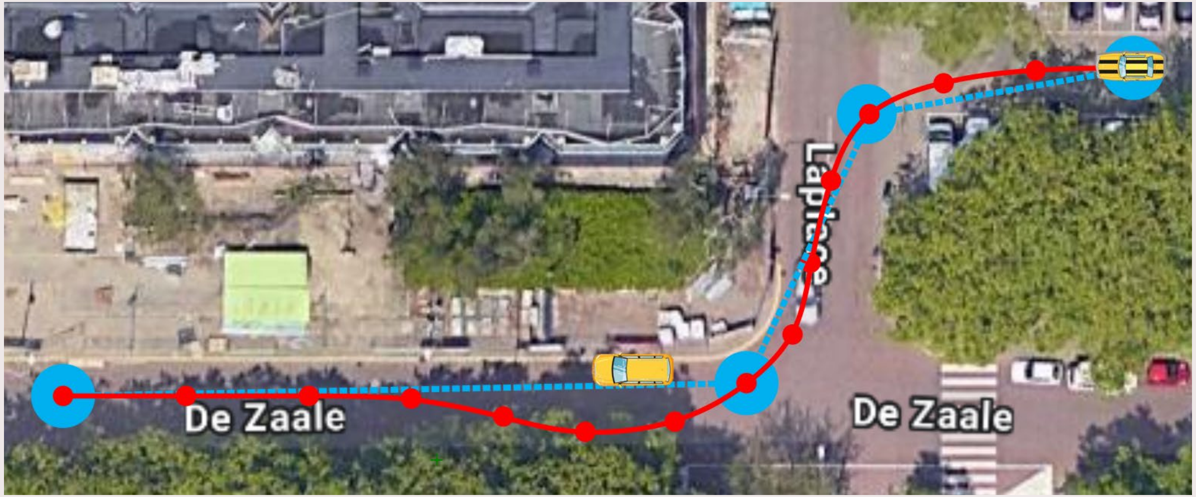
Robot navigation problem / local vs. global

- Division into global and local navigation. Why?
  1. Reduce complexity
     - Global: compute path from start to goal
     - Local: move towards the goal using the global path as a guide

Robot navigation problem / local vs. global



Global



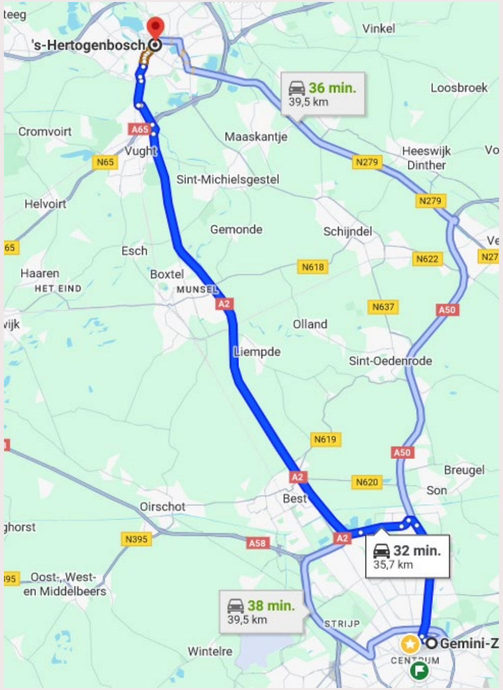
Local

TU/e

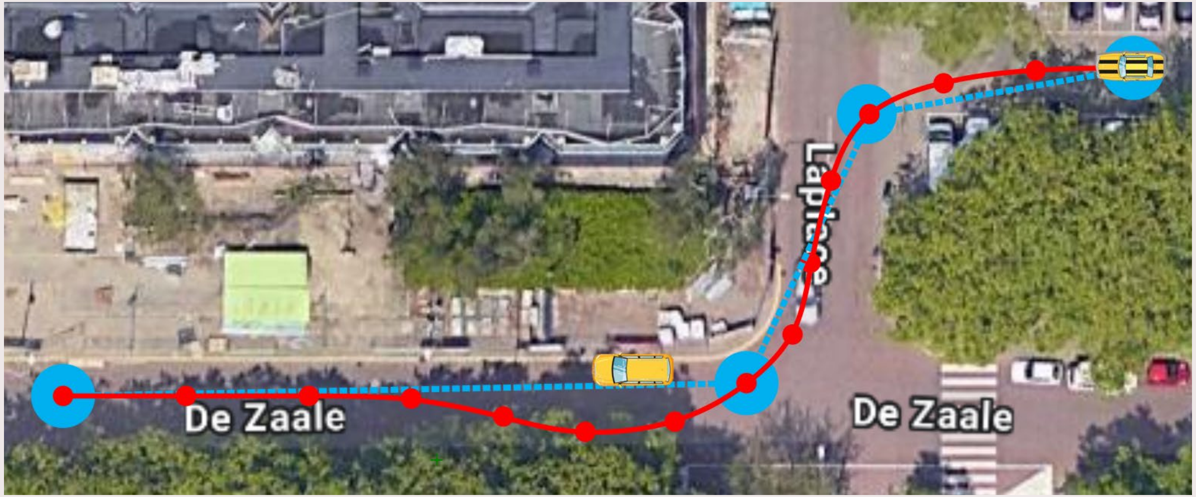
Robot navigation problem / local vs. global

- Division into global and local navigation. Why?
  1. Reduce complexity
  2. Static vs. dynamic environment
     - ➢ Global: static environment
     - ➢ Local: uncertain, dynamic environment

Robot navigation problem / local vs. global



Global



Local

TU/e

Robot navigation problem / local vs. global

- Division into global and local navigation. Why?
  1. Reduce complexity
  2. Static vs. dynamic environment
  3. Global world model often incomplete
     - More information might come with time

Robot navigation problem / local vs. global

- Division into global and local navigation. Why?
- Where is local and global?

Robot navigation problem / local vs. global

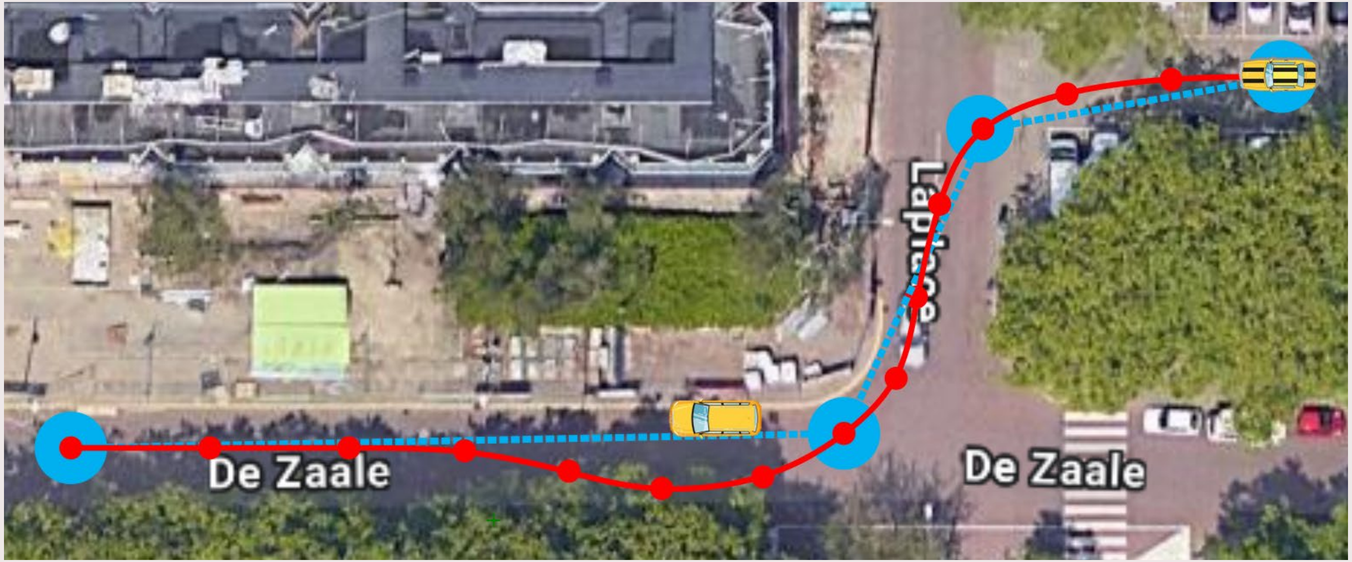
- Division into global and local navigation. Why?
- Where is local and global?
  - Problem-dependent, but in general:
    - Local: sensor-range
    - Global: map
  - **Note**: explicitly define local and global to avoid confusion!

Robot navigation problem / local vs. global

- Division into global and local navigation. Why?
- Where is local and global?
- This week: **local navigation**
  - How can we solve this?
- Next week: global navigation

Local navigation algorithms / properties

- Goal of local navigation: go from A to B, using the global path as a guide



Local navigation algorithms / properties

- Goal of local navigation: go from A to B, using the global path as a guide
- Properties of local navigation algorithms

- Completeness: finding a path if one exists
- Optimality: finding the optimal path (time, energy, distance, …)
- Computational complexity: scalability
- Robustness against a dynamic environment
- Robustness against uncertainty
- Kinematic and dynamic constraints

Local navigation algorithms / properties

- Last week’s exercise: the art of nothing crashing
  - Let the robot drive forward and let it stop before it hits anything
  - **How to go to a certain goal**?
  - We want to balance not crashing and reaching the goal
- Several approach exist, we will discuss three today

- Completeness: finding a path if one exists
- Optimality: finding the optimal path (time, energy, distance, …)
- Computational complexity: scalability
- Robustness against a dynamic environment
- Robustness against uncertainty
- Kinematic and dynamic constraints

Local navigation algorithms / examples

- Three examples
  - Artificial potential fields
  - Dynamic window approach
  - Vector field histograms

Local navigation algorithms / examples

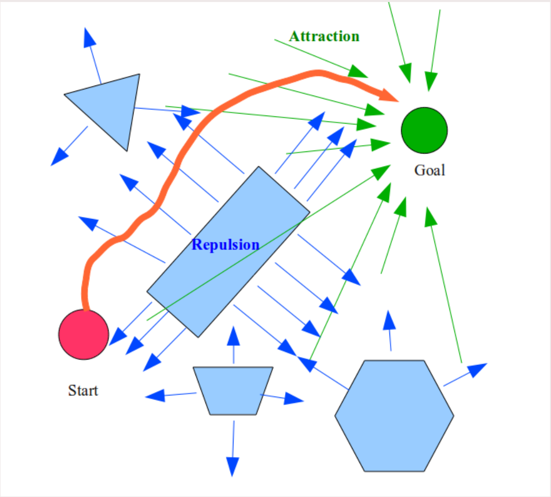
- Three examples
- Assumptions
  - A global path is available
  - Robot position is known
  - Obstacle positions are known

Local navigation algorithms / examples

- Three examples
- Assumptions
- Note that the explained algorithms directly provide **control outputs**
  - Often, a **path** is the output of a local navigation algorithm with requires a path following controller to obtain control outputs

Local navigation algorithms / artificial potential fields

- Artificial potentials
  - Attraction towards goal
  - Repulsion from obstacles
  - Think about marbles



https://sudonull.com/post/62343-What-robotics-can-teach-gaming-AI

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$ (See Chap 12.6 of [1])

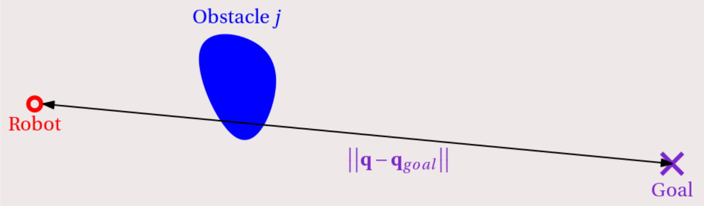
- Note
  - $\boldsymbol{q}$ is the robot configuration, in example: $\boldsymbol{q} = [x, y]$
  - $k, \rho_o > 0$
  - 'Goal' is next point of global path
  - $\|\boldsymbol{q} - \boldsymbol{q}_j^o\|$ is to the closest point of the object

[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$ (See Chap 12.6 of [1])
  - $U_{att}(\boldsymbol{q}) = \frac{1}{2} k_a \left(\left\| \boldsymbol{q} - \boldsymbol{q}_{goal} \right\| \right)^2$

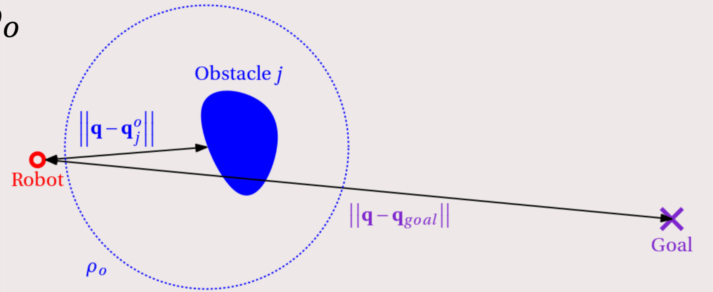
- Note
  - $\boldsymbol{q}$ is the robot configuration, in example: $\boldsymbol{q} = [x, y]$
  - $k, \rho_o > 0$
  - 'Goal' is next point of global path
  - $\left\| \boldsymbol{q} - \boldsymbol{q}_j^o \right\|$ is to the closest point of the object



[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

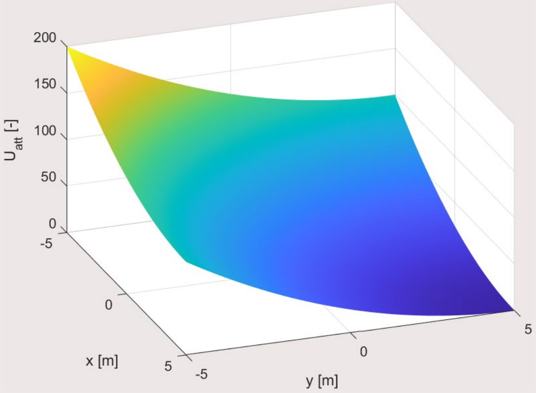
- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$ (See Chap 12.6 of [1])
  - $U_{att}(\boldsymbol{q}) = \frac{1}{2} k_a \left(\left\|\boldsymbol{q} - \boldsymbol{q}_{goal}\right\|\right)^2$
  - $U_{rep,j}(\boldsymbol{q}) = \begin{cases} \frac{1}{2} k_{rep,j} \left(\frac{1}{\left\|\boldsymbol{q}-\boldsymbol{q}_j^o\right\|} - \frac{1}{\rho_o}\right)^2 & \text{if } \left\|\boldsymbol{q} - \boldsymbol{q}_j^o\right\| \leq \rho_o \\ 0 & \text{if } \left\|\boldsymbol{q} - \boldsymbol{q}_j^o\right\| > \rho_o \end{cases}$
  - Note
    - $\boldsymbol{q}$ is the robot configuration, in example: $\boldsymbol{q} = [x, y]$
    - $k, \rho_o > 0$
    - 'Goal' is next point of global path
    - $\left\|\boldsymbol{q} - \boldsymbol{q}_j^o\right\|$ is to the closest point of the object

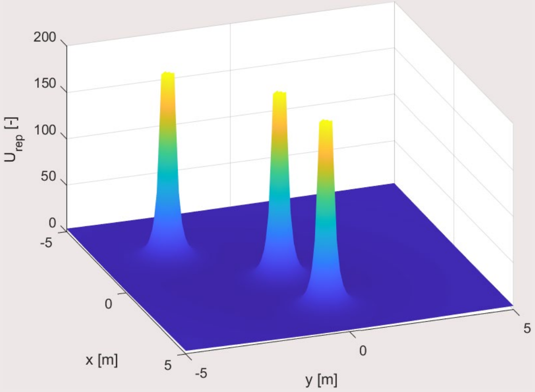


[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$ (See Chap 12.6 of [1])





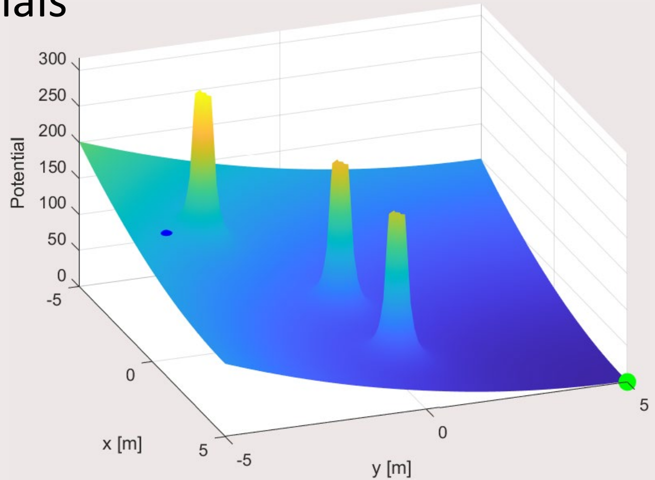
- Starting point: $[-2, -4]$
- Goal point: $[5,5]$
- 3 obstacles: $[-2, -3], [0{,}5, -0.5], [3{,}0]$

[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$
- Total potential field is the sum of individual potentials

$$U(\boldsymbol{q}) = U_{att}(\boldsymbol{q}) + \sum_{j=1}^{n} U_{rep,j}(\boldsymbol{q})$$



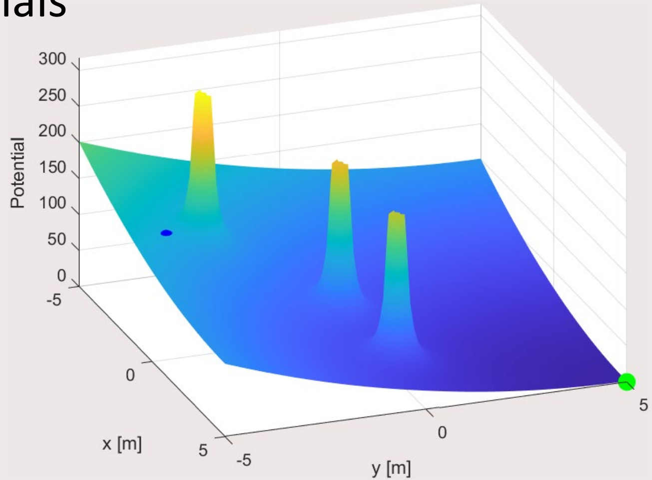
- Starting point: $[-2, -4]$
- Goal point: $[5,5]$
- 3 obstacles: $[-2,-3]$, $[0{,}5,-0.5]$, $[3,0]$

[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$
- Total potential field is the sum of individual potentials
- The artificial force acting on the robot is then

$$F(\boldsymbol{q}) = -\nabla U(\boldsymbol{q})$$



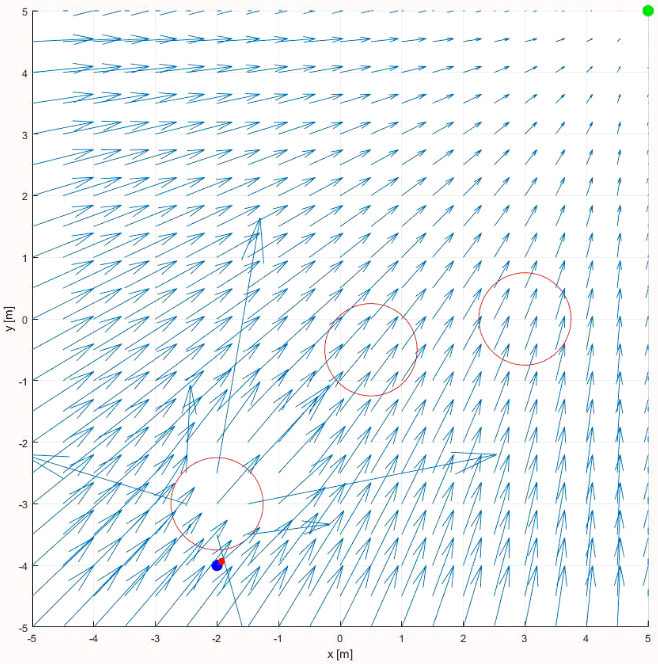
[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$
- Total potential field is the sum of individual potentials
- The artificial force acting on the robot is then
- How to use that force?
  - Point mass: $\ddot{\boldsymbol{q}} = F(\boldsymbol{q})$
  - Desired velocity: $\dot{\boldsymbol{q}} = F(\boldsymbol{q})$

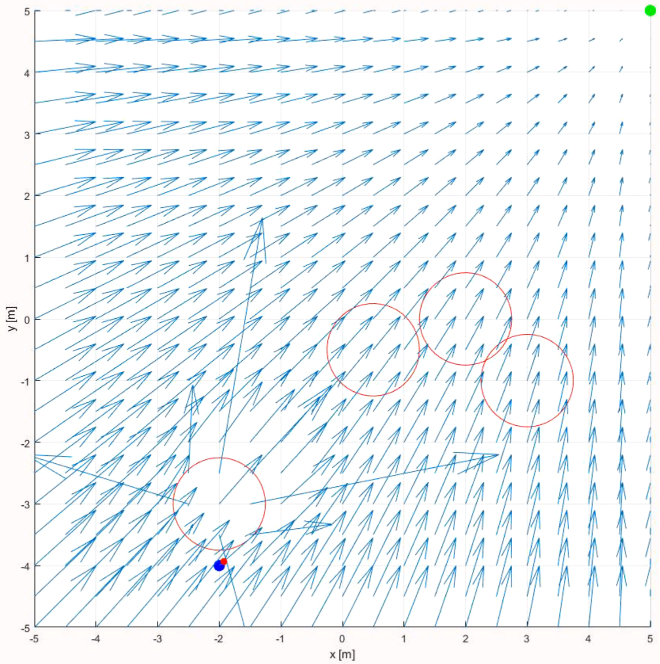
[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

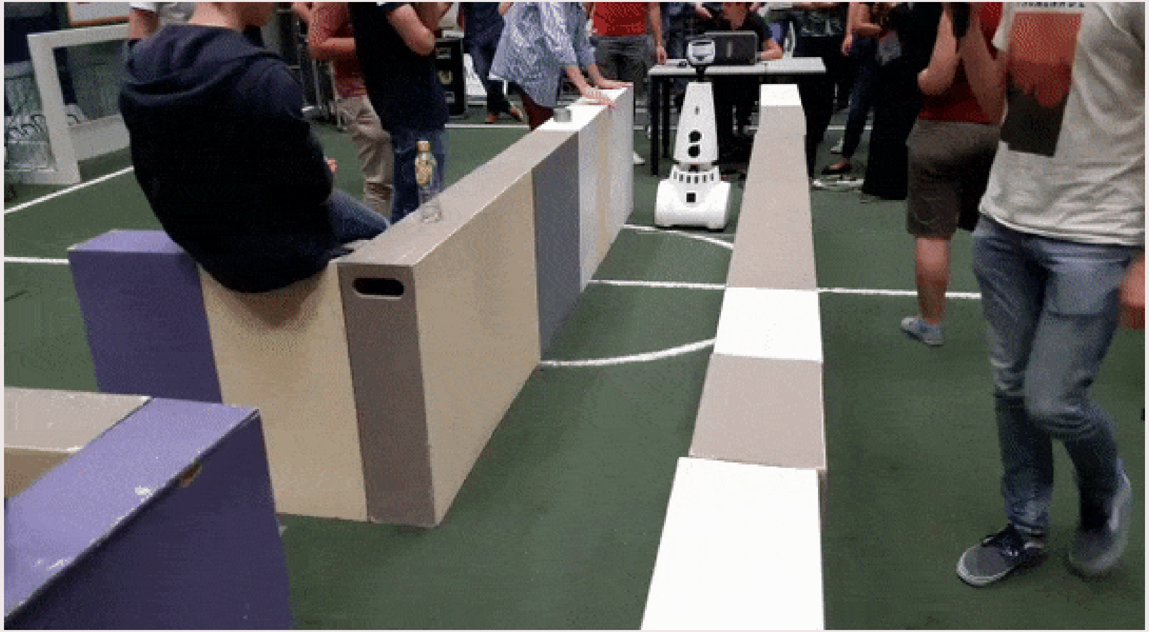


TU/e

Local navigation algorithms / artificial potential fields



Local navigation algorithms / artificial potential fields



EMC 2017 – Group 10

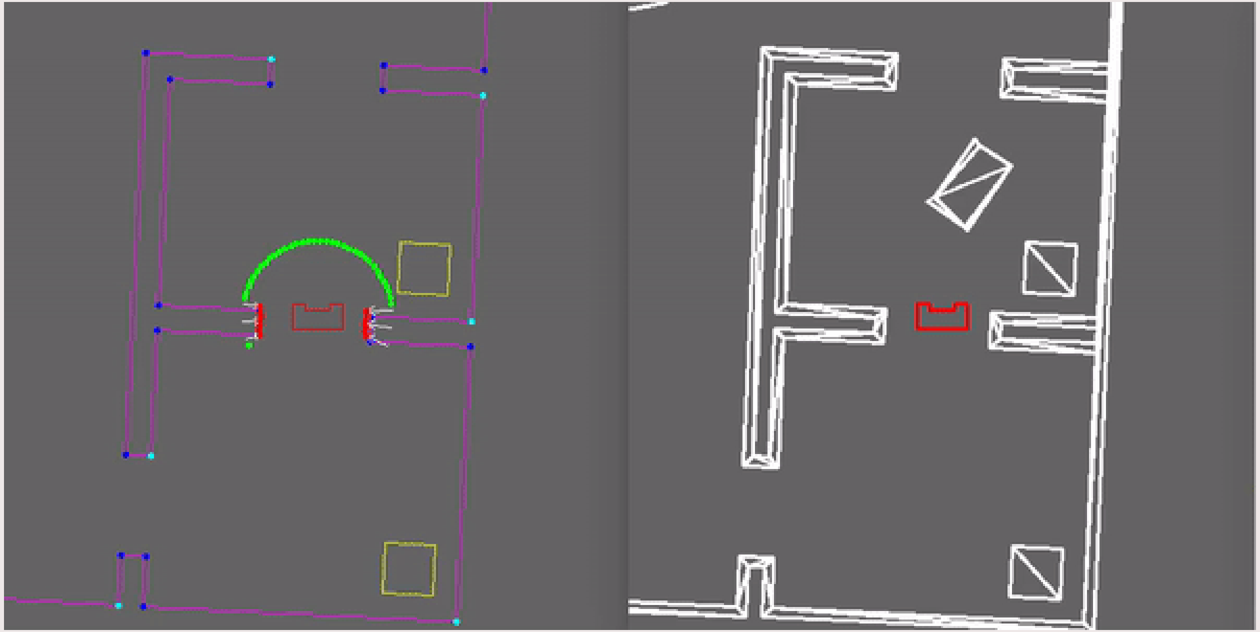
TU/e

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$
- Total potential field is the sum of individual potentials
- The artificial force acting on the robot is then
- How to use that force?
- In the simulation videos we know everything… **How to do it in real life**?

[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields



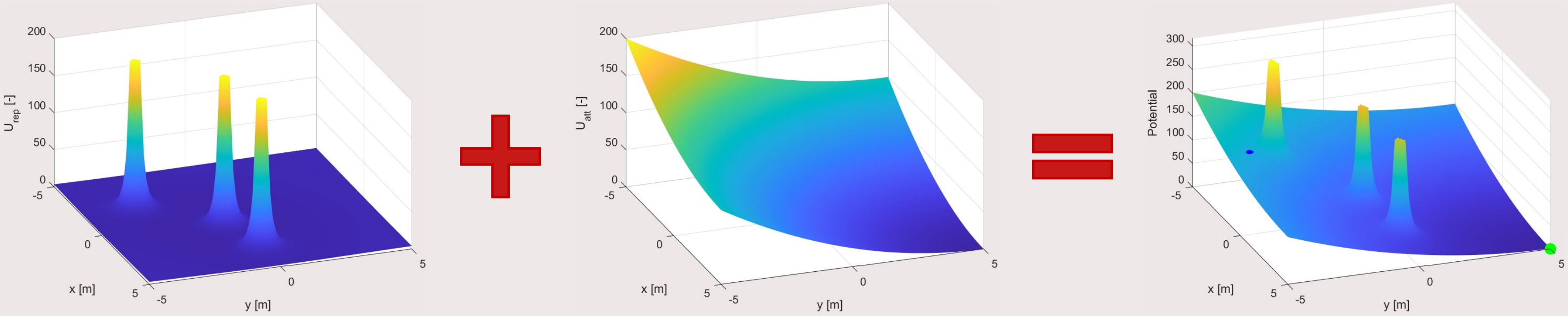
Simulation - MRC 2019 – Group 2

Local navigation algorithms / artificial potential fields

- Artificial potentials
- Amplitude based on distance to object $\boldsymbol{q}_j^o$ and goal $\boldsymbol{q}_{goal}$
- Total potential field is the sum of individual potentials
- The artificial force acting on the robot is then
- How to use that force?
- In the simulation videos we know everything… **How to do it in real life**?
    - How to represent obstacles from laser points?
    - Include size of the robot

[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / artificial potential fields

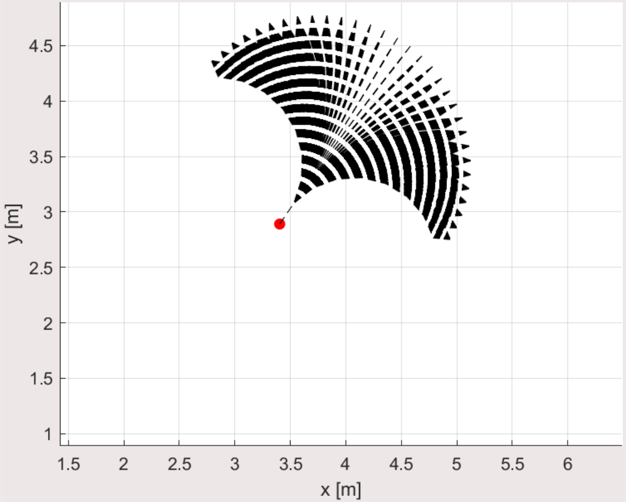


Questions?

[1] B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

Local navigation algorithms / dynamic window approach

- Reactive collision avoidance based on robot dynamics
  - Intuition: certain velocity during certain time, see where we end and select most optimal



D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t, where (v, ω) have to be

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t, where (v, ω) have to be
  - Possible: velocities are limited by robot's dynamics

$$V_s = \{v, \omega | v \in [v_{min}, v_{max}] \land \omega \in [\omega_{min}, \omega_{max}]\}$$

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t, where (v, ω) have to be
  - Possible: velocities are limited by robot's dynamics
  - Admissible: robot can stop before reaching closest obstacle

$$V_a = \left\{ v, \omega \middle| v \leq \sqrt{2d(v,\omega)\dot{v}_b} \land \omega \leq \sqrt{2d(v,\omega)\dot{\omega}_b} \right\}$$

$\dot{v}_b$ and $\dot{\omega}_b$ are maximum deceleration values

$d(v, \omega)$ is distance to closest object

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

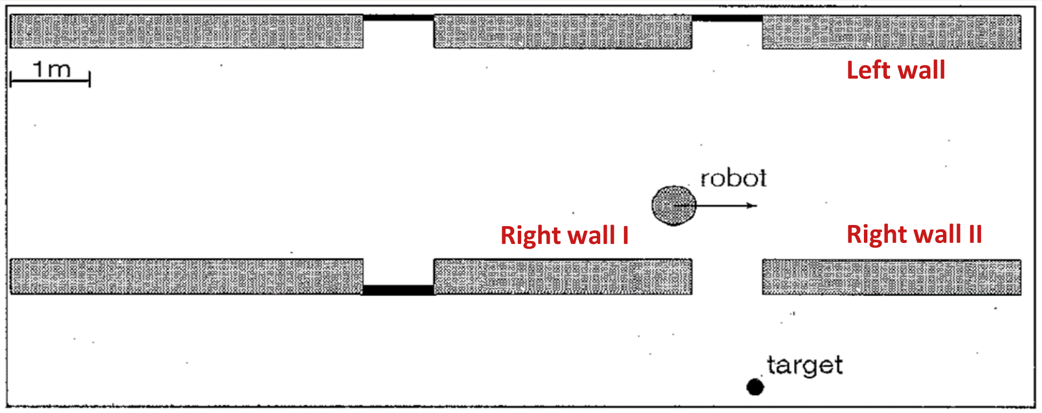
- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t, where (v, ω) have to be
  - Possible: velocities are limited by robot's dynamics
  - Admissible: robot can stop before reaching closest obstacle
  - Reachable: velocity and acceleration constraints (dynamic window)

$$V_d = \{v, \omega | v \in [v_a - \dot{v}t, v_a + \dot{v}t] \wedge \omega \in [\omega_a - \dot{\omega}t, \omega_a + \dot{\omega}t]\}$$

v_a and w_a are actual velocities
$\dot{v}$ and $\dot{\omega}$ are acceleration values

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

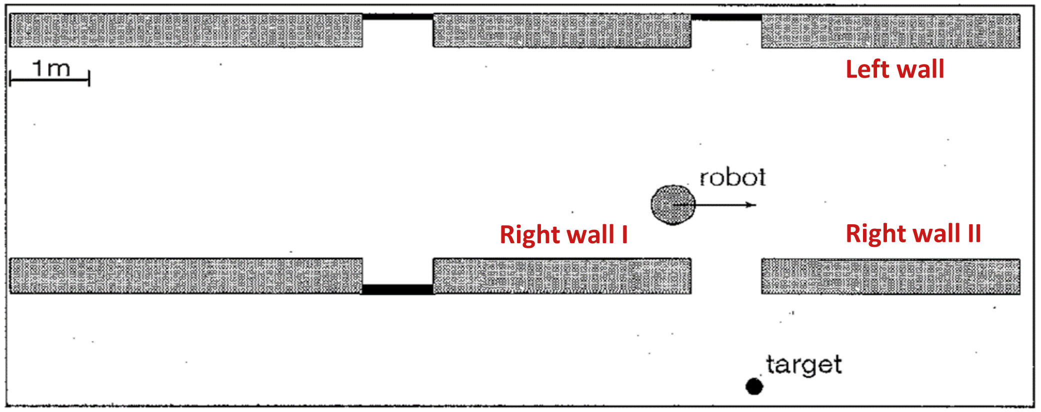


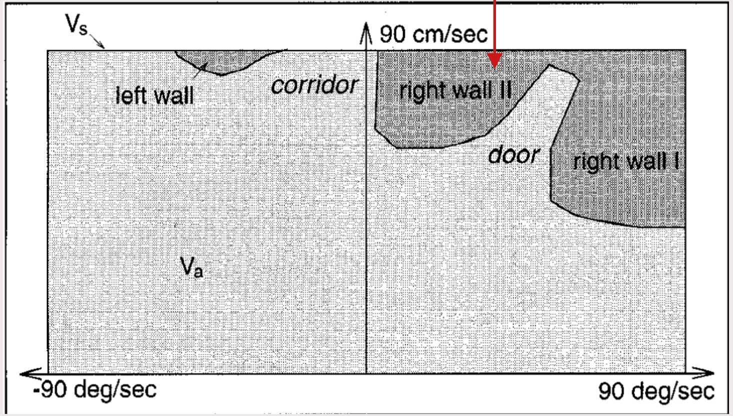
V_a: admissible velocities
V_r: reachable velocities
V_s: velocity search space

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

Dark shade: non-admissible velocities





V_a: admissible velocities
V_r: reachable velocities
V_s: velocity search space

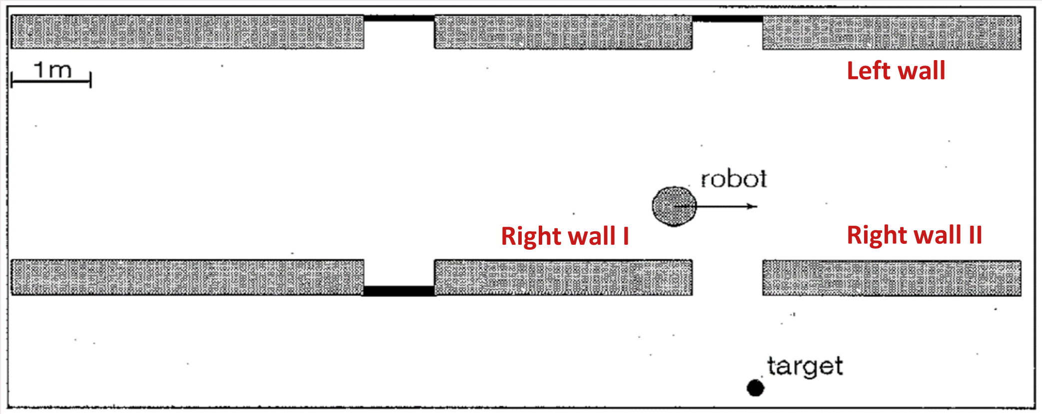
$$V_s = \{v, \omega | v \in [v_{min}, v_{max}] \wedge \omega \in [\omega_{min}, \omega_{max}]\}$$

$$V_a = \left\{v, \omega | v \leq \sqrt{2d(v,\omega)\dot{v}_b} \wedge \omega \leq \sqrt{2d(v,\omega)\dot{\omega}_b}\right\}$$

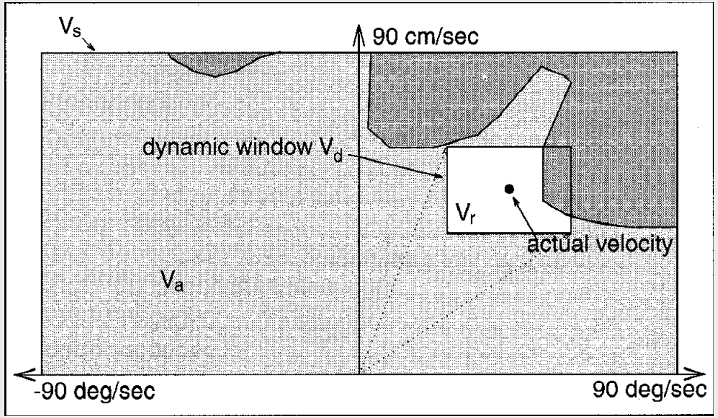
Here $\dot{v}_b = 50\ \text{cm}/s^2, \dot{\omega}_b = 60\ deg/s^2$

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach



V_a: admissible velocities
V_r: reachable velocities
V_s: velocity search space



$$V_d = \{v, \omega | v \in [v_a - \dot{v}\Delta t, v_a + \dot{v}\Delta t] \wedge \omega \in [\omega_a - \dot{\omega}\Delta t, \omega_a + \dot{\omega}\Delta t]\}$$

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

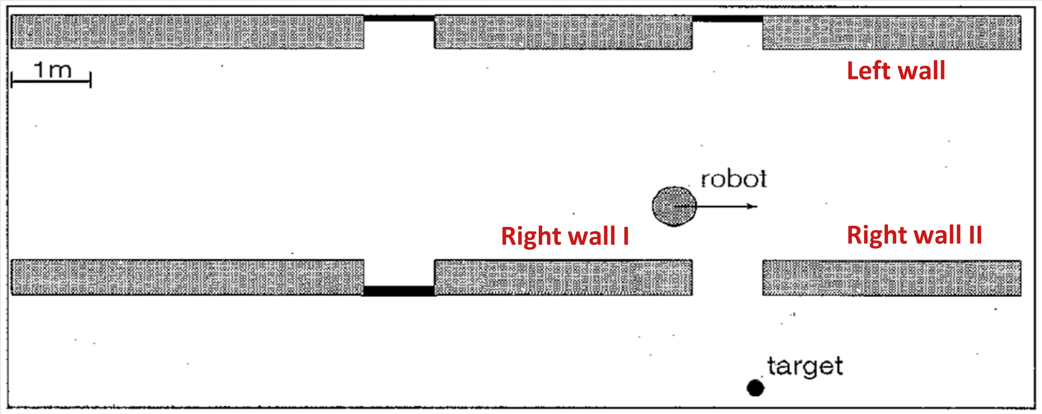
- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t: possible, admissible, reachable
- Generate search space
  - Intersection of V_s, V_a and V_d provides search space V_r

$$V_r = V_s \cap V_a \cap V_d$$

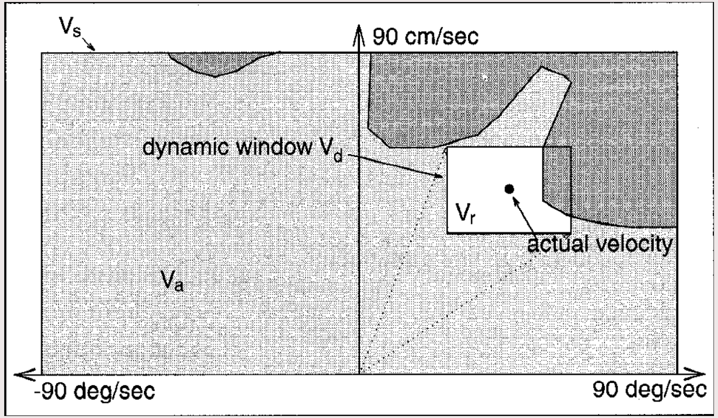
$\rightarrow$ gives $\left(v_{range}, \omega_{range}\right) \in V_r$ at each time step

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach



V_a: admissible velocities
V_r: reachable velocities
V_s: velocity search space



$\rightarrow V_r = V_s \cap V_a \cap V_d$ (white area)
$\rightarrow (v_{range}, \omega_{range}) \in V_r$

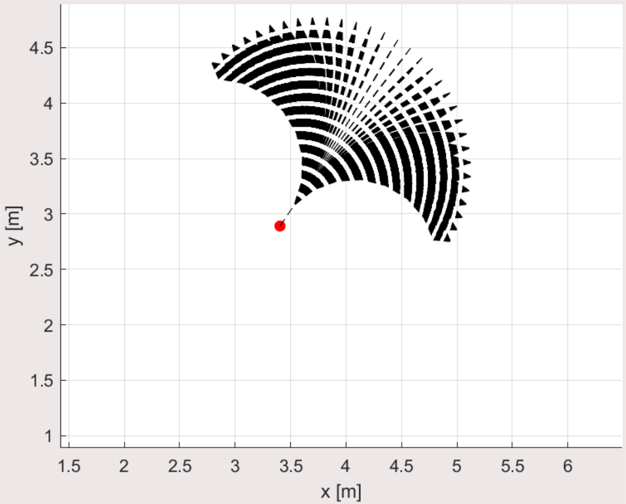
D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t: possible, admissible, reachable
- Generate search space

$x(0), y(0)$ and $\theta(0)$ are current position

$$
\begin{aligned}
&\text{for } i = 0{:}N \\
&\quad \text{for } j = 1{:}\,\mathrm{len}\left(v_{range}\right) \\
&\qquad \text{for } k = 1{:}\,\mathrm{len}\left(\omega_{range}\right) \\
&\qquad\qquad x(i+1) = x(i) + v_{range}(j) \cdot \cos\left(\theta(i)\right) \\
&\qquad\qquad y(i+1) = y(i) + \Delta t \cdot v_{range}(j) \cdot \sin\left(\theta(i)\right) \\
&\qquad\qquad \theta(i+1) = \theta(i) + \Delta t \cdot \omega_{range}(k)
\end{aligned}
$$



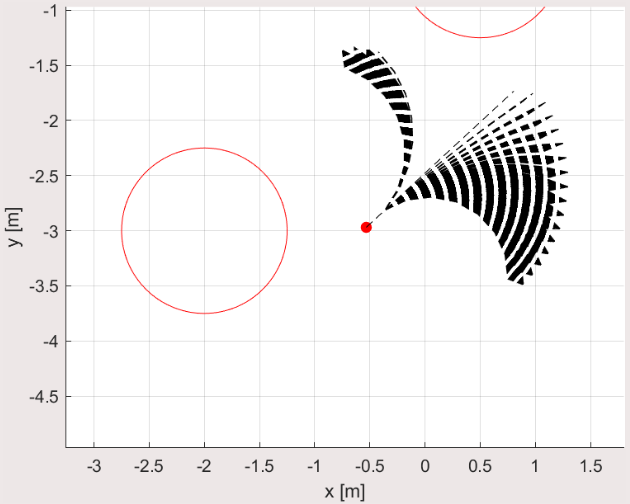
D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t: possible, admissible, reachable
- Generate search space

$x(0), y(0)$ and $\theta(0)$ are current position

$$
\begin{aligned}
&\text{for } i = 0{:}N\\
&\quad \text{for } j = 1{:}\operatorname{len}(v_{range})\\
&\quad\quad \text{for } k = 1{:}\operatorname{len}(\omega_{range})\\
&\quad\quad\quad x(i+1) = x(i) + v_{range}(j)\cdot\cos(\theta(i))\\
&\quad\quad\quad y(i+1) = y(i) + \Delta t\cdot v_{range}(j)\cdot\sin(\theta(i))\\
&\quad\quad\quad \theta(i+1) = \theta(i) + \Delta t\cdot \omega_{range}(k)
\end{aligned}
$$



D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

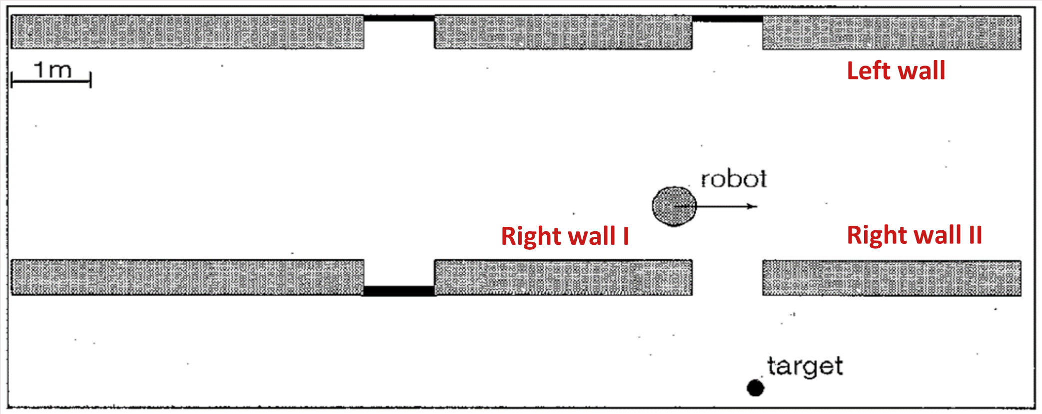
- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t: possible, admissible, reachable
- Generate search space
- Maximize objective function G

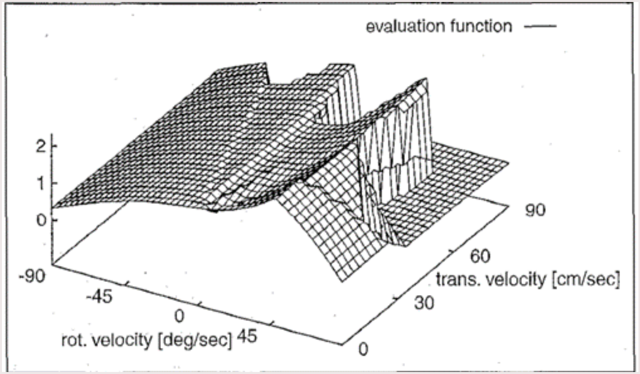
$$G(v, \omega) = \sigma\big(k_h h(v, \omega) + k_d d(v, \omega) + k_s s(v, \omega)\big)$$

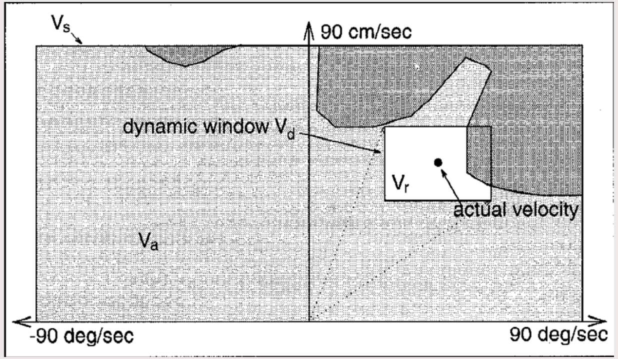
  - $h(v, \omega)$: target heading towards goal
  - $d(v, \omega)$: distance to closest obstacle on trajectory
  - $s(v, \omega)$: forward velocity

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

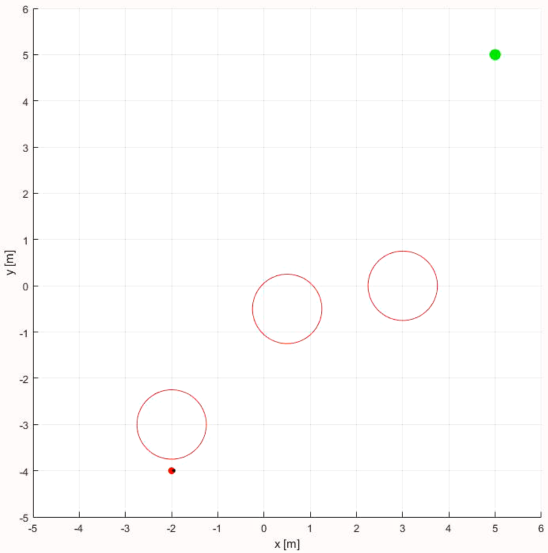


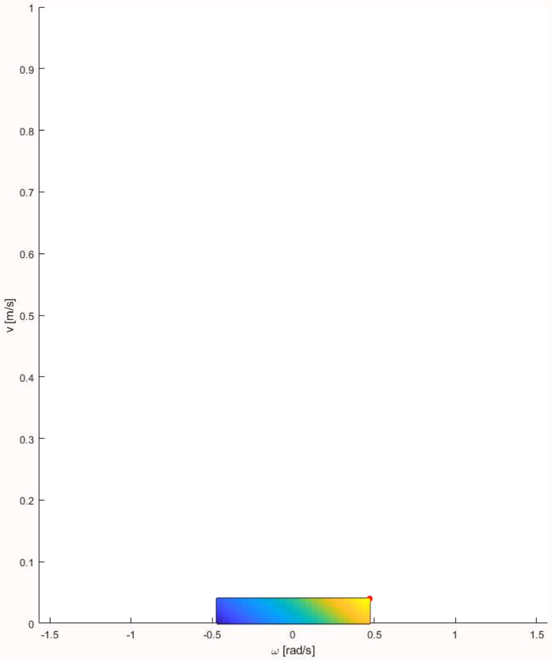




D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

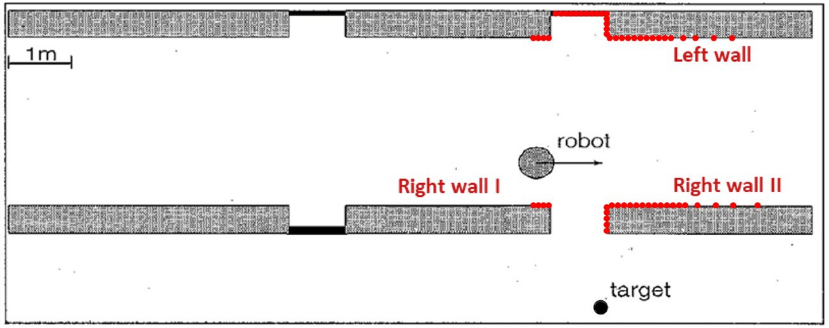




D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

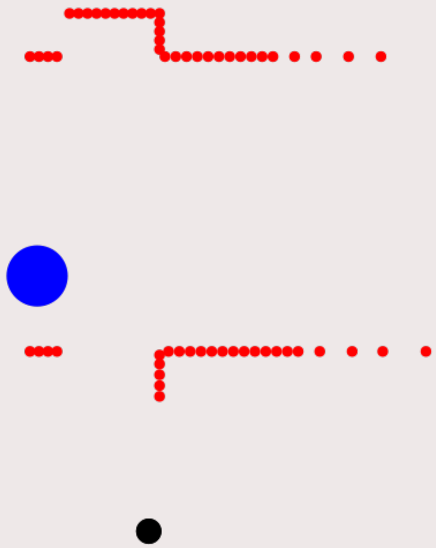
- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t: possible, admissible, reachable
- Generate search space
- Maximize objective function G
- Again, we have all information in simulation videos…



D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach

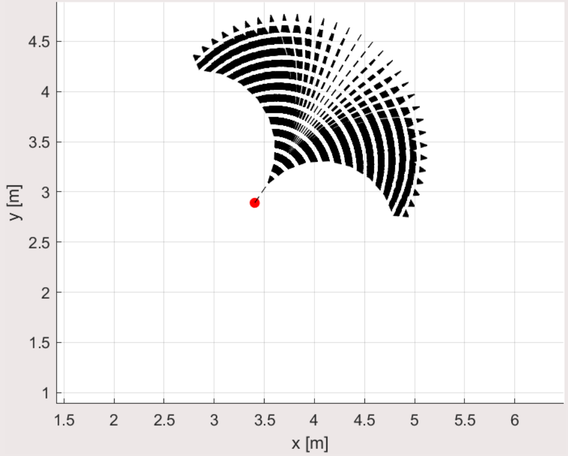
- How to represent the obstacles?
- Available information:
  - Laser range points
  - Trajectory from discretized velocities might fall between two points
- Also, incorporate the size of the robot
  - In the video, robot is a point mass



D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

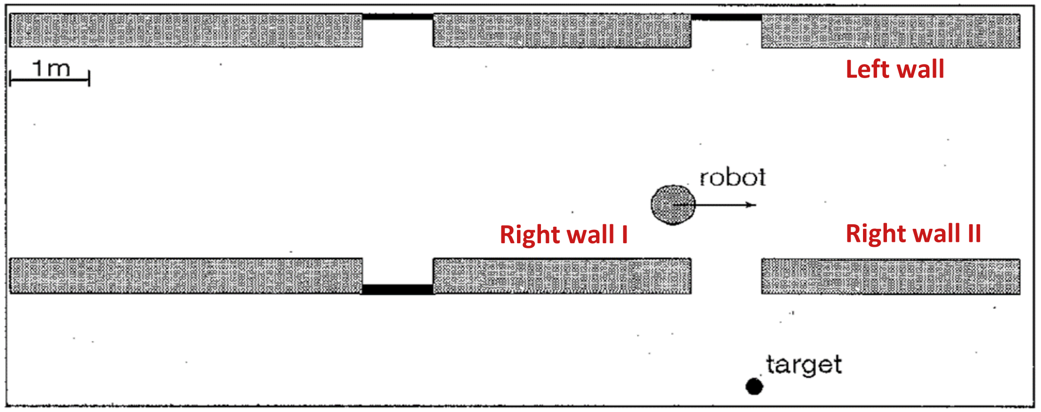
Local navigation algorithms / dynamic window approach

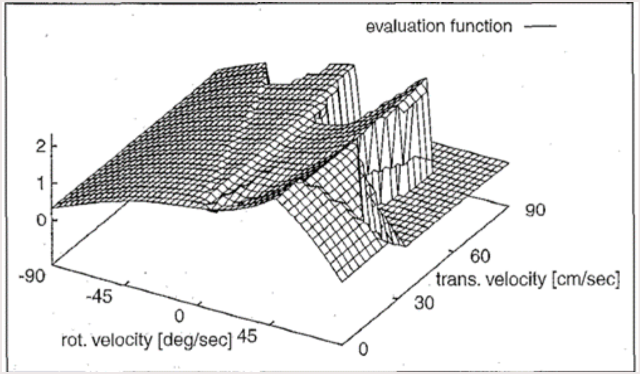
- Reactive collision avoidance based on robot dynamics
- Consider velocities (v, ω) during t: possible, admissible, reachable
- Generate search space
- Maximize objective function G
- Again, we have all information in simulation videos…
- Implementation
  - How to check if a path is valid?
  - How discretize v_{range} and ω_{range}?
  - How to account for robot size?

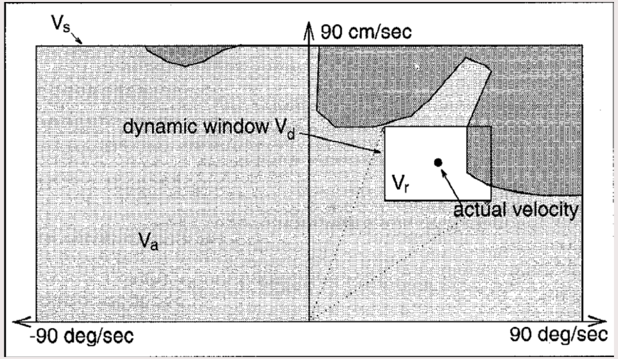


D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

Local navigation algorithms / dynamic window approach



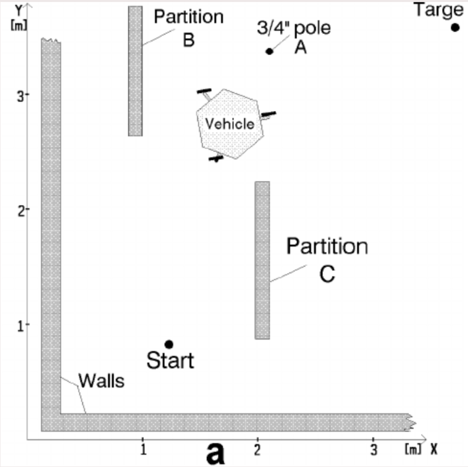


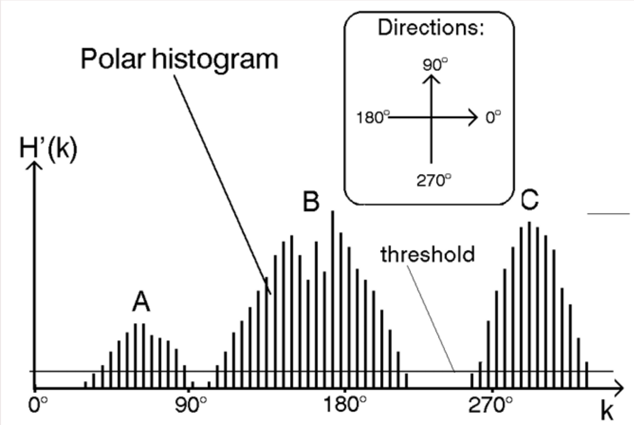


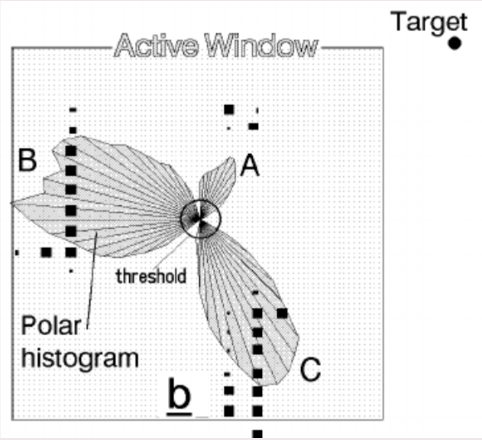
Questions?

Local navigation algorithms / vector field histograms

- Treat objects as vectors in a 2D Cartesian histogram grid, and create a polar histogram to determine possible ‘open spaces’ to get to the goal

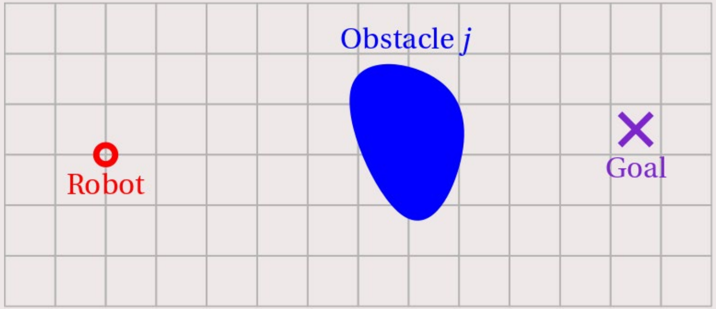






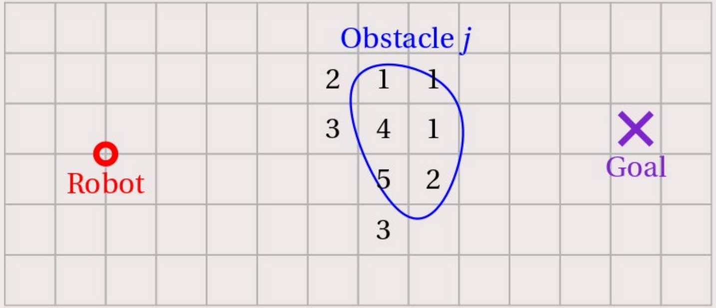
Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid



Local navigation algorithms / vector field histograms

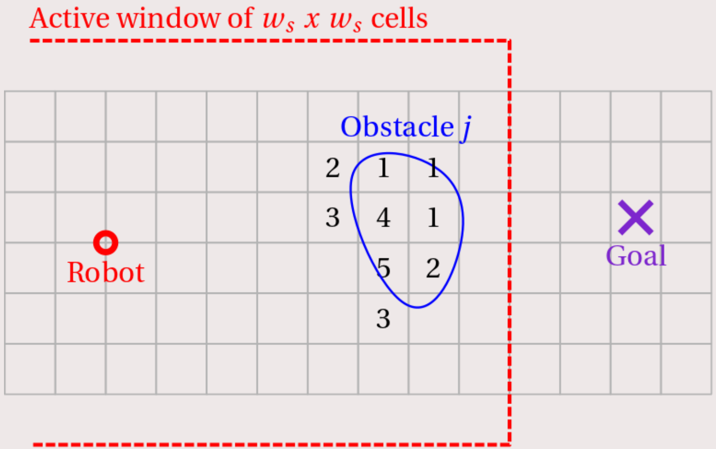
- 2D Cartesian histogram grid
  - each cell holds a certainty (or confidence) value $c_{i,j}$ of that cell containing an obstacle



TU/e

Local navigation algorithms / vector field histograms

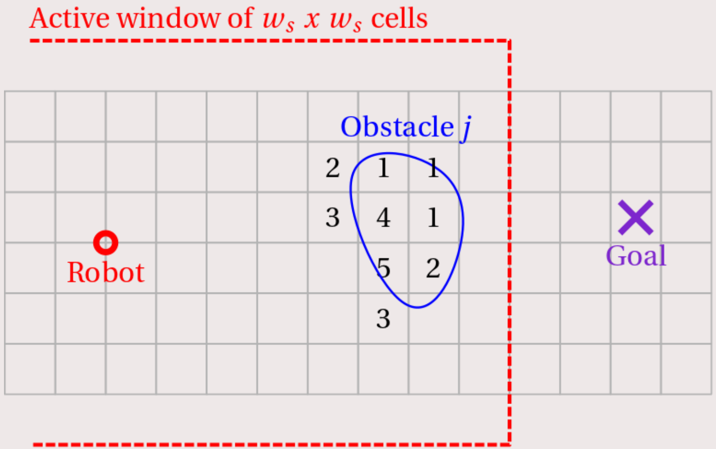
- 2D Cartesian histogram grid
  - each cell holds a certainty (or confidence) value $c_{i,j}$ of that cell containing an obstacle
  - Active window



Note that the active window should be square and centered around robot, drawing is purely for visualization of the approach

Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
  - each cell holds a certainty (or confidence) value $c_{i,j}$ of that cell containing an obstacle
  - Active window
  - Each active cell is treated as obstacle vector with
    - direction $\beta_{i,j} = \text{atan2}(y_j - y_0, x_i - x_0)$
    - magnitude $m_{i,j} = c_{i,j}^2(a - bd_{i,j})$
      - Choose a, b such that $a - bd_{max} = 0$
      - $d_{max} = \frac{\sqrt{2}}{2}(w_s - 1)$
      - see [1] for further explanation on the values of a and b



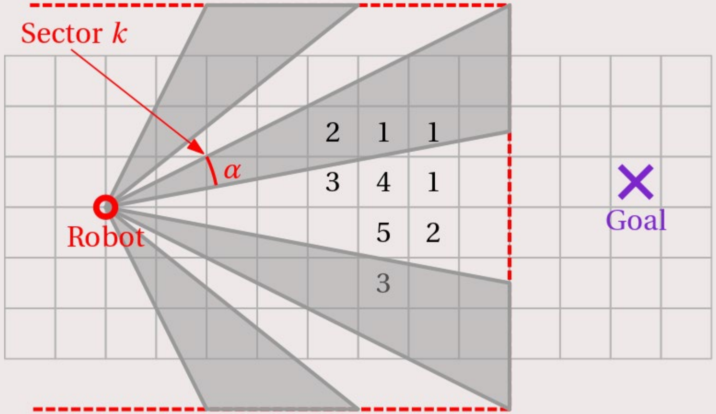
[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
  - Sector k corresponds to angular resolution α

$$\alpha = \frac{360°}{n}$$

n is an integer, $k = 0,1,2,\ldots,n-1$

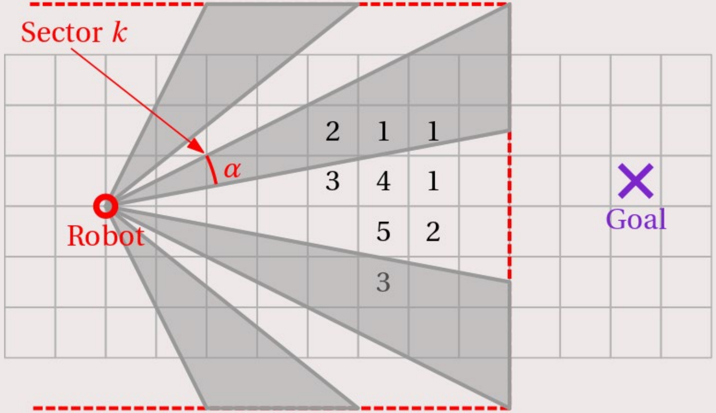


[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
  - Sector k corresponds to angular resolution α
  - Link between each cell $c_{i,j}$ and k

$$k = \mathrm{int}\left(\frac{\beta_{i,j}}{\alpha}\right)$$



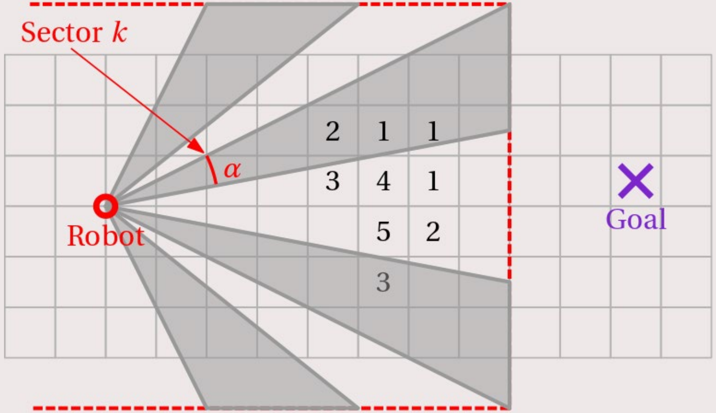
[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
  - Sector k corresponds to angular resolution α
  - Link between each cell $c_{i,j}$ and k
  - For each sector k, polar obstacle density h_k is

$$h_k = \sum_{i,j} m_{i,j}$$

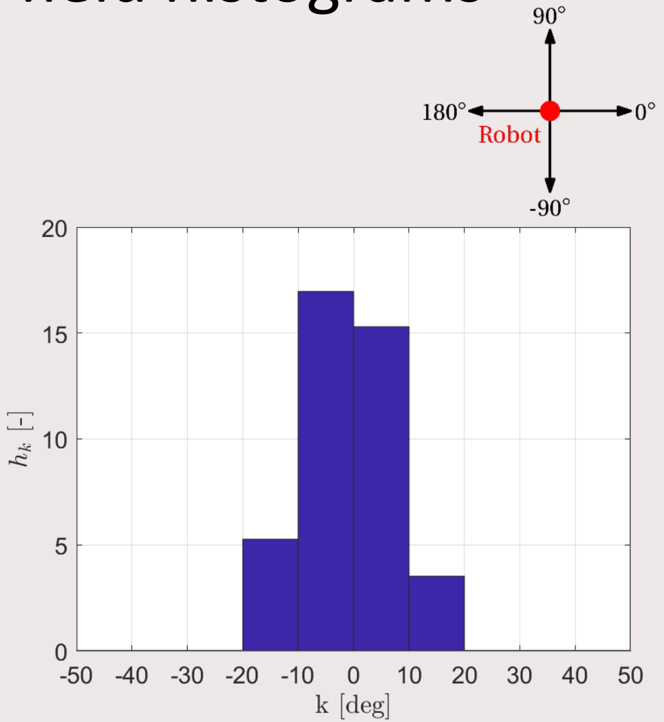
Note: needs smoothing due to discrete map, see [1]



[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

Local navigation algorithms / vector field histograms

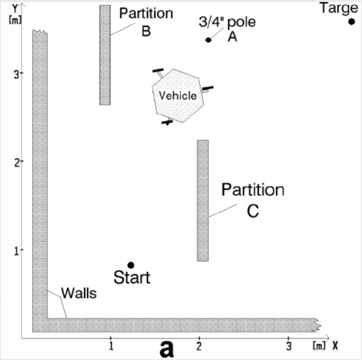
- 2D Cartesian histogram grid
- Polar histogram
  - Sector k corresponds to angular resolution α
  - Link between each cell $c_{i,j}$ and k
  - For each sector k, polar obstacle density h_k
  - Resulting histogram
    - Note that the figure only shows $[-50°, 50°]$, but the histogram is actually $[-180°, 180°)$
    - Note that no smoothing is applied

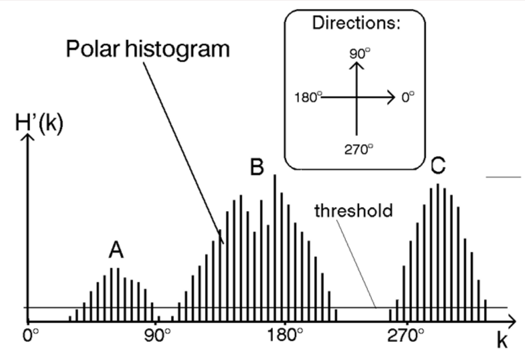


[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
  - Smoothed polar histogram $H'(k)$ [1]

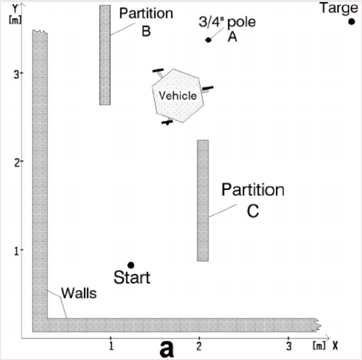


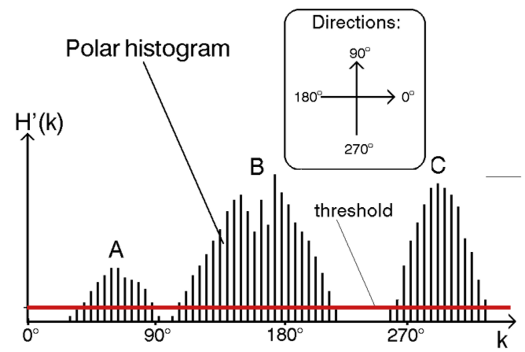


[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
  - Smoothed polar histogram $H'(k)$ [1]
  - Candidate valleys: $H'(k)$ below threshold





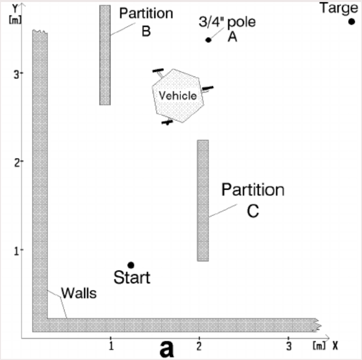
[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

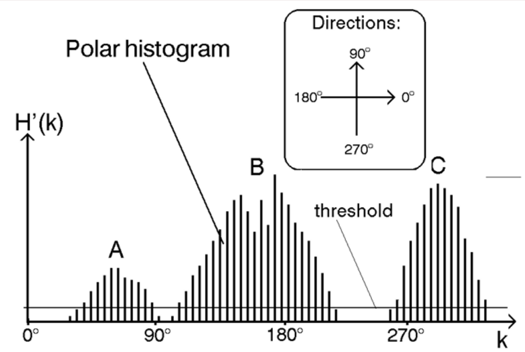
Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
  - Smoothed polar histogram $H'(k)$ [1]
  - Candidate valleys: $H'(k)$ below threshold
  - Angle θ is the middle of candidate valley

$$\theta = \frac{1}{2}\alpha(k_l + k_r)$$

k_l and k_r are left and right boundary of selected valley





[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

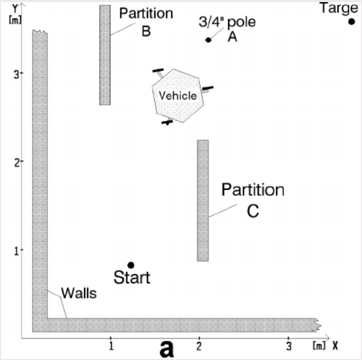
Local navigation algorithms / vector field histograms

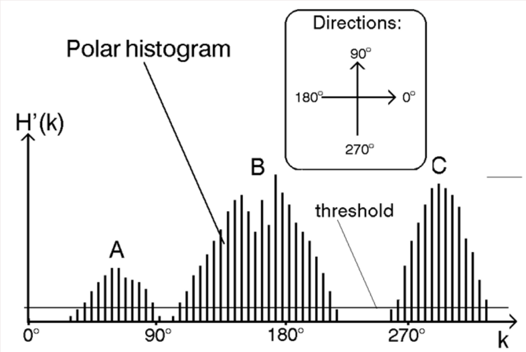
- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
  - Smoothed polar histogram $H'(k)$ [1]
  - Candidate valleys: $H'(k)$ below threshold
  - Angle θ is the middle of candidate valley

$$\theta = \frac{1}{2}\alpha(k_l + k_r)$$

k_l and k_r are left and right boundary of selected valley

  - Select the valley with closest match to goal direction





[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

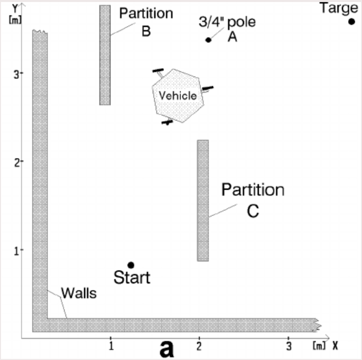
Local navigation algorithms / vector field histograms

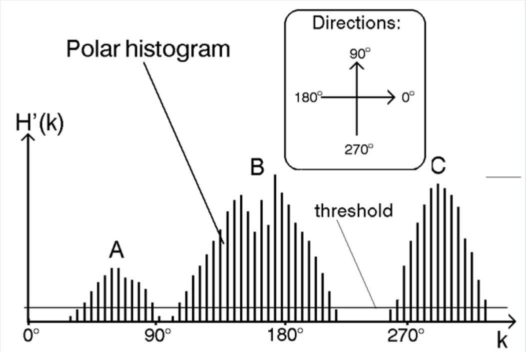
- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
  - Smoothed polar histogram $H'(k)$ [1]
  - Candidate valleys: $H'(k)$ below threshold
  - Angle θ is the middle of candidate valley

$$\theta = \frac{1}{2}\alpha(k_l + k_r)$$

k_l and k_r are left and right boundary of selected valley

  - Select the valley with closest match to goal direction
  - Controller (e.g., PI) to align robot with goal direction





[1] J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.

Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
- Velocity control
  - Anticipatory reduction: $v' = V_{max}\left(1 - \frac{1}{h_m}\min(h_c', h_m)\right)$

h_c': obstacle density in current direction of travel
h_m: empirically determined constant to obtain sufficient speed reduction

TU/e

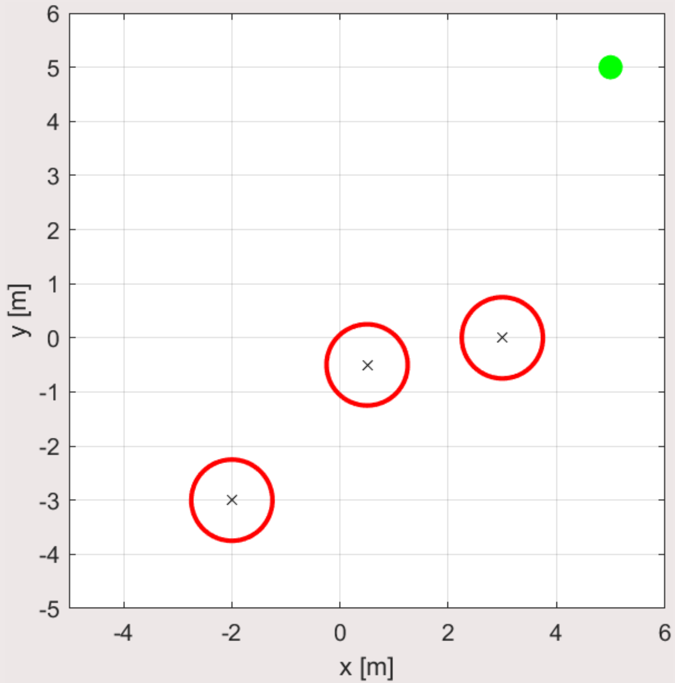
Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
- Velocity control
  - Anticipatory reduction: $v' = V_{max}\left(1 - \frac{1}{h_m}\min(h'_c, h_m)\right)$
  - Steering speed reduction: $v = v'\left(1 - \frac{\dot{\theta}}{\dot{\theta}_{max}}\right) + V_{min}$

h'_c: obstacle density in current direction of travel
h_m: empirically determined constant to obtain sufficient speed reduction
$\dot{\theta}$: steering rate

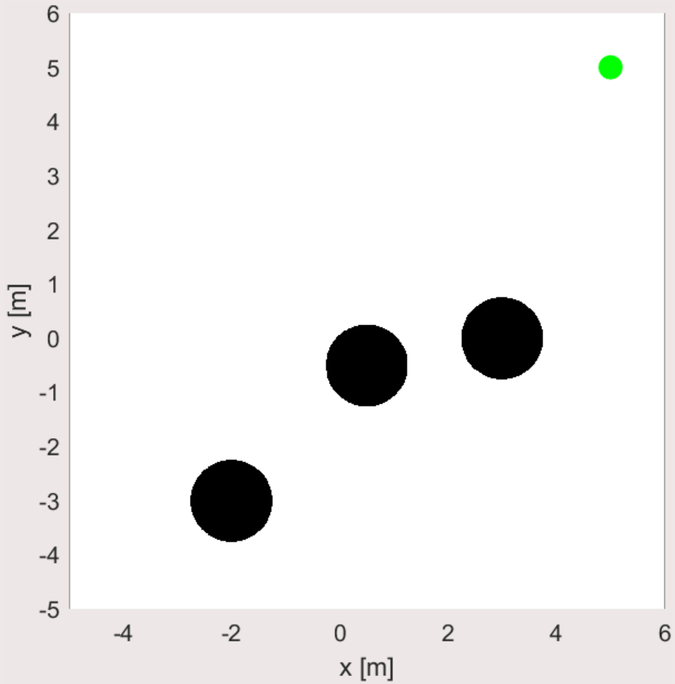
Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
- Velocity control
- Example
  - Grid world map to create histogram grid

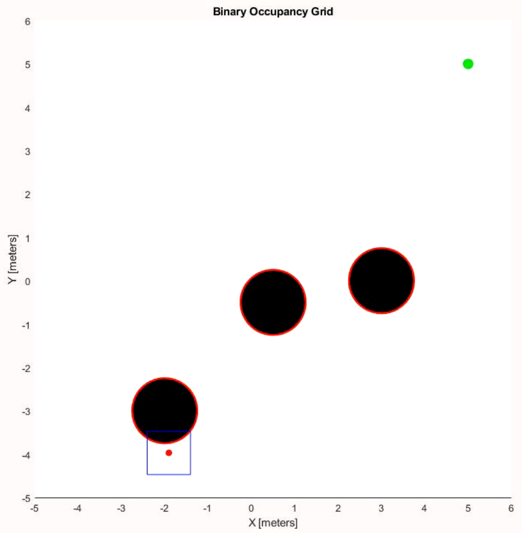


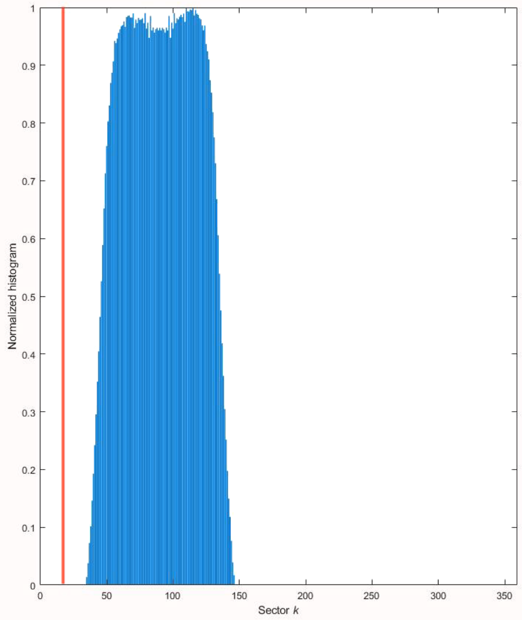
Local navigation algorithms / vector field histograms

- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
- Velocity control
- Example
  - Grid world map to create histogram grid
  - Assumed that obstacle position is fully known



Local navigation algorithms / vector field histograms

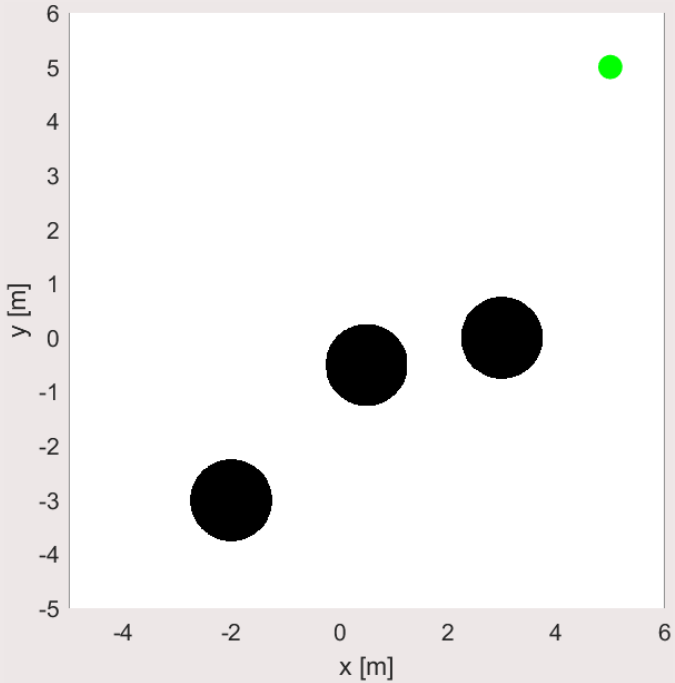




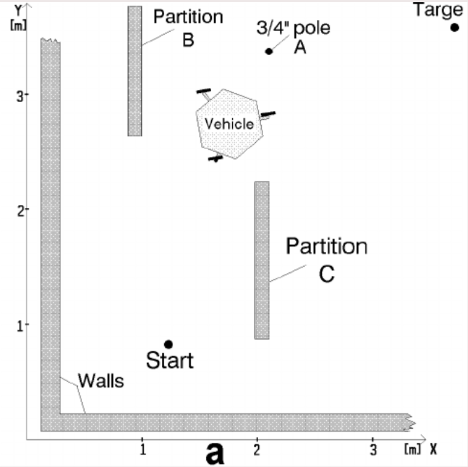
Orientation is in world frame

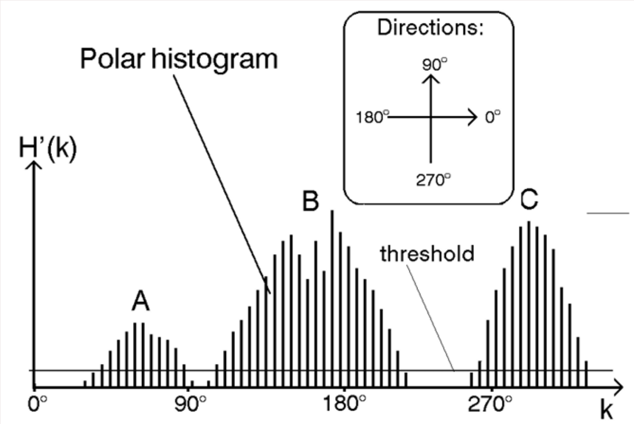
Local navigation algorithms / vector field histograms

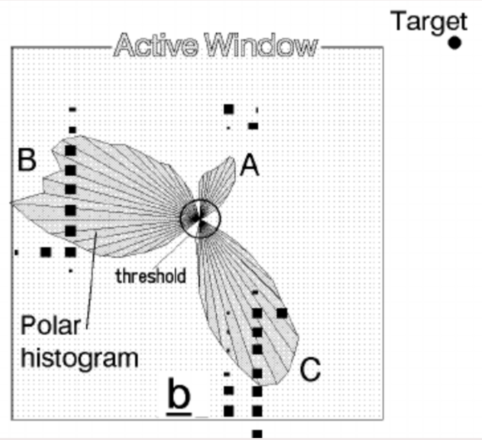
- 2D Cartesian histogram grid
- Polar histogram
- Steering direction
- Velocity control
- Implementation considerations
  - Again, think about the size of the robot
  - How to create the Cartesian histogram grid from sensor data?
  - What is the desired angle if there are no obstacles in the active window?



Local navigation algorithms / vector field histograms







Questions?

Local navigation algorithms / comparison of discussed approaches

- Artificial Potential Fields
  - Repulsion from objects and attraction to goal
  - Simple and computationally efficient
  - Suffers from local minimal and not optimal paths

Local navigation algorithms / comparison of discussed approaches

- Artificial Potential Fields
  - Repulsion from objects and attraction to goal
  - Simple and computationally efficient
  - Suffers from local minimal and not optimal paths
- Dynamic Window Approach
  - Generate feasible action space based on robot dynamics within time horizon
  - Considers robot dynamics → collision-free and feasible trajectories
  - Requires accurate sensor data, might struggle with densely-populated environments

Local navigation algorithms / comparison of discussed approaches

- Artificial Potential Fields
  - Repulsion from objects and attraction to goal
  - Simple and computationally efficient
  - Suffers from local minimal and not optimal paths
- Dynamic Window Approach
  - Generate feasible action space based on robot dynamics within time horizon
  - Considers robot dynamics → collision-free and feasible trajectories
  - Requires accurate sensor data, might struggle with densely-populated environments
- Vector Field Histograms
  - Create polar histogram of confidence on object location
  - Computationally efficient, robust to noisy sensor data
  - Can struggle with narrow passages and sharp corners

Local navigation algorithms / other possible approaches

- Optimization based
  - Minimize objective function limited by constraints and system dynamics to find the ‘optimal’ path or trajectory
  - Objective function:
    - Distance/time to goal,
    - Smoothness of trajectory,
    - Comfort (acceleration/jerk),
    - Safety related.

$$\min \int_0^T J\big(x(t), u(t)\big)$$

subject to

$$x(0) = x_0$$
$$\dot{x}(t) = f\big(x(t), u(t)\big)$$
$$g\big(x(t), u(t)\big) \leq 0$$
$$\underline{u} \leq u(t) \leq \overline{u}$$
$$\underline{x} \leq x(t) \leq \overline{x}$$

Local navigation algorithms / other possible approaches

- Optimization based
- Learning based
  - Relies heavily on training sensor data,
  - Train a learning model (e.g., neural network) to
    - Predict behaviour of environment
    - Detect obstacles
    - Decision-making
  - Based on real-life sensor data, create necessary output

https://www.youtube.com/watch?v=FwT4TSRsiVw

Local navigation algorithms / other possible approaches

- Optimization based
- Learning based
- **Note:**
  - We have explained three approaches from a wide range of possibilities
  - In the exercises, you are allowed to implement approaches not treated in this lecture
  - But note that more complex is not necessarily better..
  - Additionally, note that the explained algorithms directly provide control outputs

Footnote: world representation

- All sensor info treated the same
- In more complex environments different objects should be treated differently based on their semantic context
  - E.g., keep more distance to humans.

Recap

- What is the robot navigation problem?
    - Find a feasible path or trajectory from a given initial pose (A) to the desired final pose (B)
- What is the goal of local navigation?
    - Go from A to B using the global path as a guide
- Local navigation algorithms: properties
- Local navigation algorithms: examples
    - Artificial potential fields
    - Dynamic window approach
    - Vector field histogram
    - Optimization and learning based methods

Assignment

- Divide your group into two (equal sized) groups
- Enable your robot to drive through a corridor to a goal position by implementing **two different local navigation algorithms** (one by each subgroup)
- Answer the provided questions, provide videos of simulations and testing on the field, and upload your code (with comments!)
- Final remark:
  - You will use one of the algorithms in the final challenge
  - Create a **function for each algorithm** (which use the same input + output) to enable easy implementation and testing

Literature

S. M. LaValle, "Planning Algorithms," Cambridge University Press, Cambridge, 2006, doi: 10.1017/CBO9780511546877.

B. Siciliano and O. Khatib, Eds., "Springer Handbook of Robotics," Springer, Berlin, Heidelberg, 2008, ISBN: 978-3-540-23957-4.

B. Siciliano, L. Sciavicco, L. Villani, and G. Oriolo, "Robotics: Modelling, Planning and Control," Springer Publishing Company, Incorporated, 2010

D. Fox, W. Burgard and S. Thrun, "The dynamic window approach to collision avoidance," in *IEEE Robotics & Automation Magazine*, vol. 4, no. 1, pp. 23-33, March 1997, doi: 10.1109/100.580977.

J. Borenstein and Y. Koren, "The vector field histogram-fast obstacle avoidance for mobile robots," in IEEE Transactions on Robotics and Automation, vol. 7, no. 3, pp. 278-288, June 1991, doi: 10.1109/70.88137.