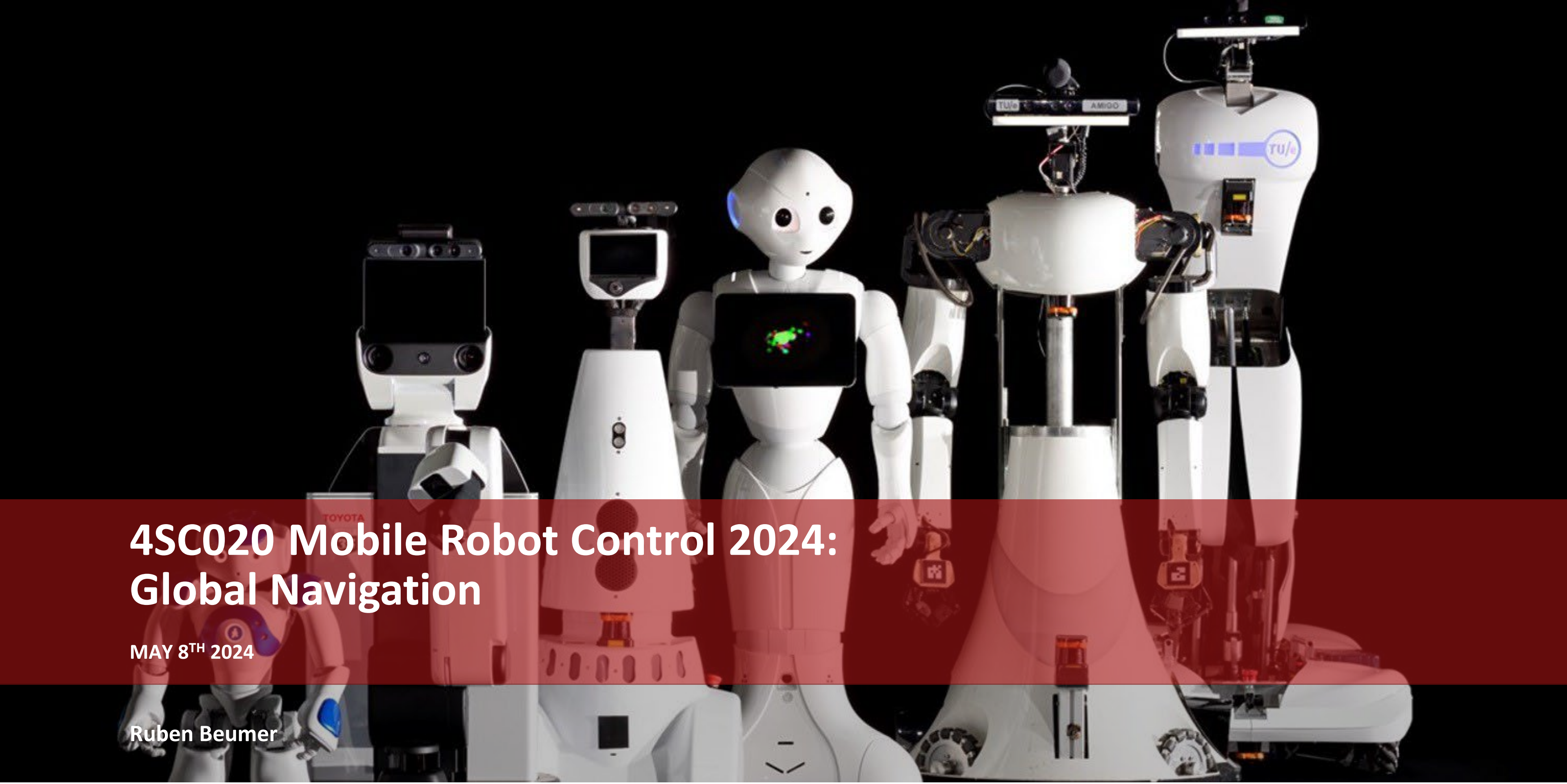

# 4SC020 Mobile Robot Control 2024: Global Navigation

MAY 8TH 2024

Ruben Beumer

Mechanical Engineering, Robotics

EINDHOVEN UNIVERSITY OF TECHNOLOGY

# Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
- Introduction to assignment

# Outline

- Recap local navigation & intro global navigation
  - Robot navigation problem
    - Global vs local navigation
  - Global navigation problem
  - Motion planning algorithms: specifications and properties
- Map representations
- From map to graph
- Path planning in graphs
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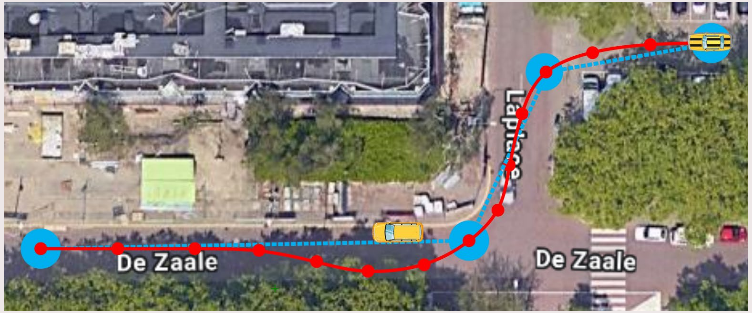
- Introduction to assignment

# **Recap & intro** / Robot navigation problem

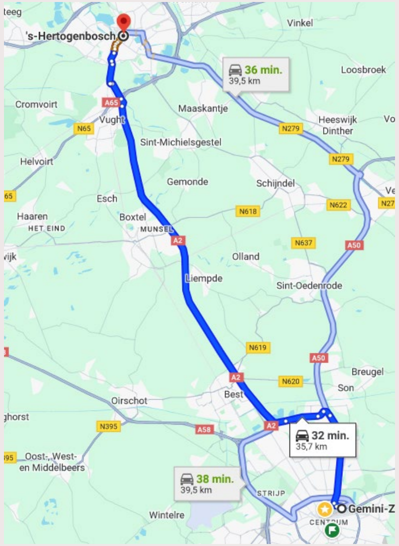
- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)

# Recap & intro / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
  - Global: compute path from start to goal
  - Local: execute local part of global path while satisfying constraints




Local




Global

# **Recap & intro** / Robot navigation problem

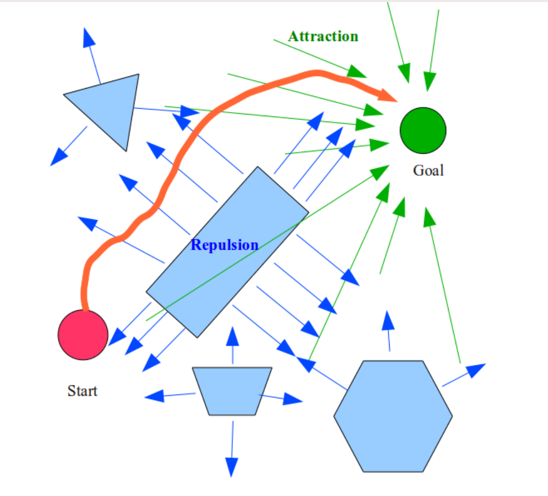
- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
  - Global: compute path from start to goal
  - Local: execute local part of global path while satisfying constraints
  - Reasons:
    - Reduce complexity
    - Static vs dynamic environment
    - Global world model often incomplete or unavailable

# **Recap & intro** / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms

# Recap & intro / Robot navigation problem

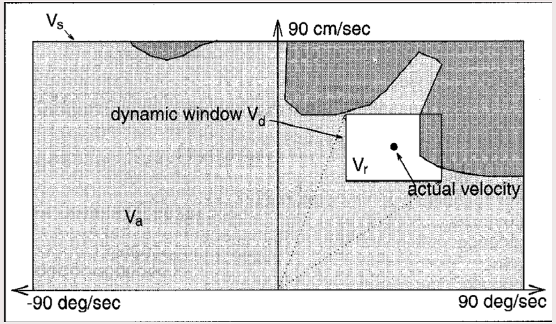
- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms
  - Artificial potential fields

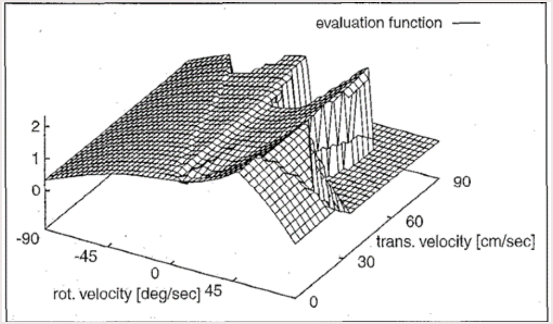



https://sudonull.com/post/62343-What-robotics-can-teach-gaming-AI

# **Recap & intro** / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms
  - Artificial potential fields
  - Dynamic window approach



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- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
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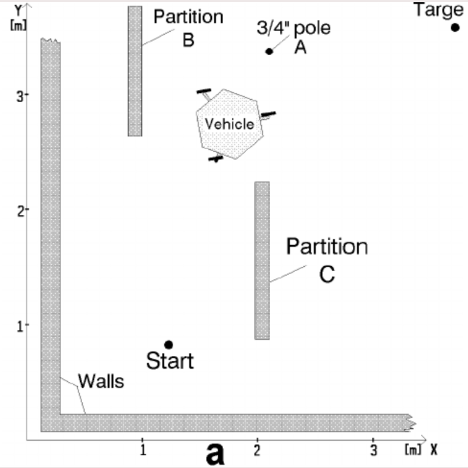
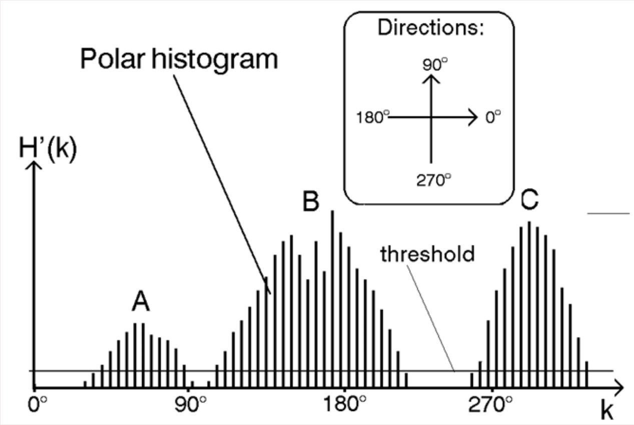
  - Dynamic window approach
  - Vector field histograms



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- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms
  - Artificial potential fields
  - Dynamic window approach
  - Vector field histograms
  - Optimization- and learning-based methods

# **Recap & intro** / Robot navigation problem

- Goal: find a path or trajectory from a given initial pose (A) to the desired final pose (B)
- Division into global and local navigation
- Local navigation algorithms

Questions?

# **Recap & intro** / Global navigation problem

- What is the global navigation problem?
  - Find a feasible path from A to B based on your current knowledge

- How does it complement local navigation?
  - Global path gives the direction to progress. Local navigation follows this direction safely, taking into account local objects

- What are the requirements?

# Recap & intro / Motion planning algorithms: specs & properties

- Completeness: finding a path if one exists
- Optimality: finding the optimal path (time, energy, distance, …)
- Computational complexity: scalability
- Robustness against a dynamic environment
- Robustness against uncertainty
- Kinematic and dynamic constraints

# Recap & intro / Motion planning algorithms: specs & properties

- Completeness: finding a path if one exists
- Optimality: finding the optimal path (time, energy, distance, …)
- Computational complexity: scalability

Also important for global planning!

- Robustness against a dynamic environment
- Robustness against uncertainty
- Kinematic and dynamic constraints

Mainly handled by local planning



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# Outline

- Recap local navigation & intro global navigation
- **Map representations**
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
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# Map representations

We often discretize the map to make the problem more manageable

Grid-based (equidistant cells)

Cell-based

Graph-based

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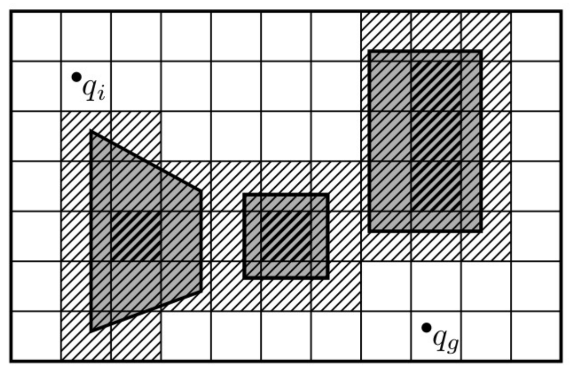


Coenen, S.A.M. (2012). Motion Planning for Mobile Robots - A Guide. Master's thesis

Cell-based

Graph-based

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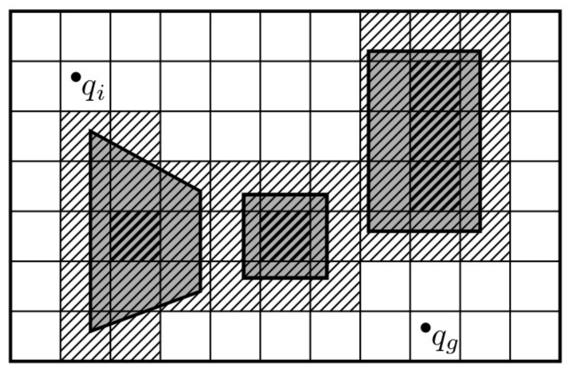


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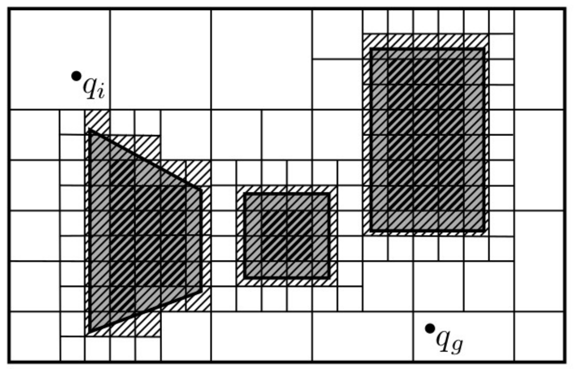


Graph-based

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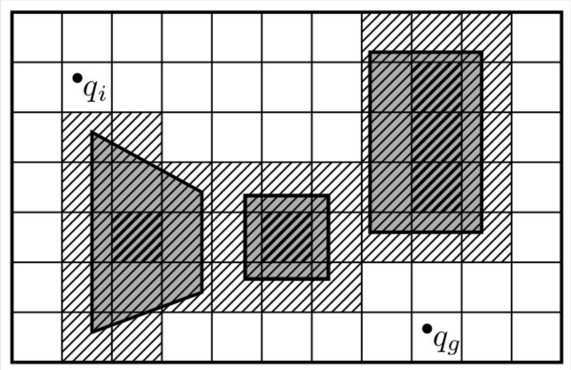


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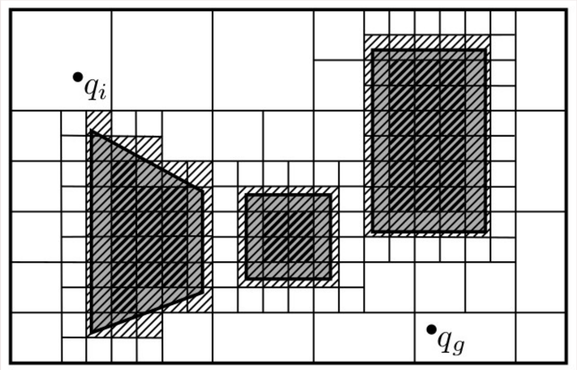


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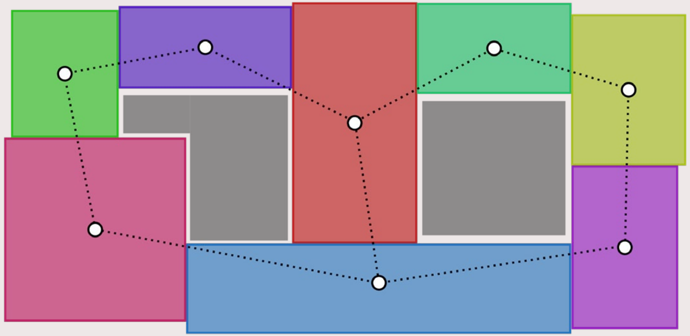


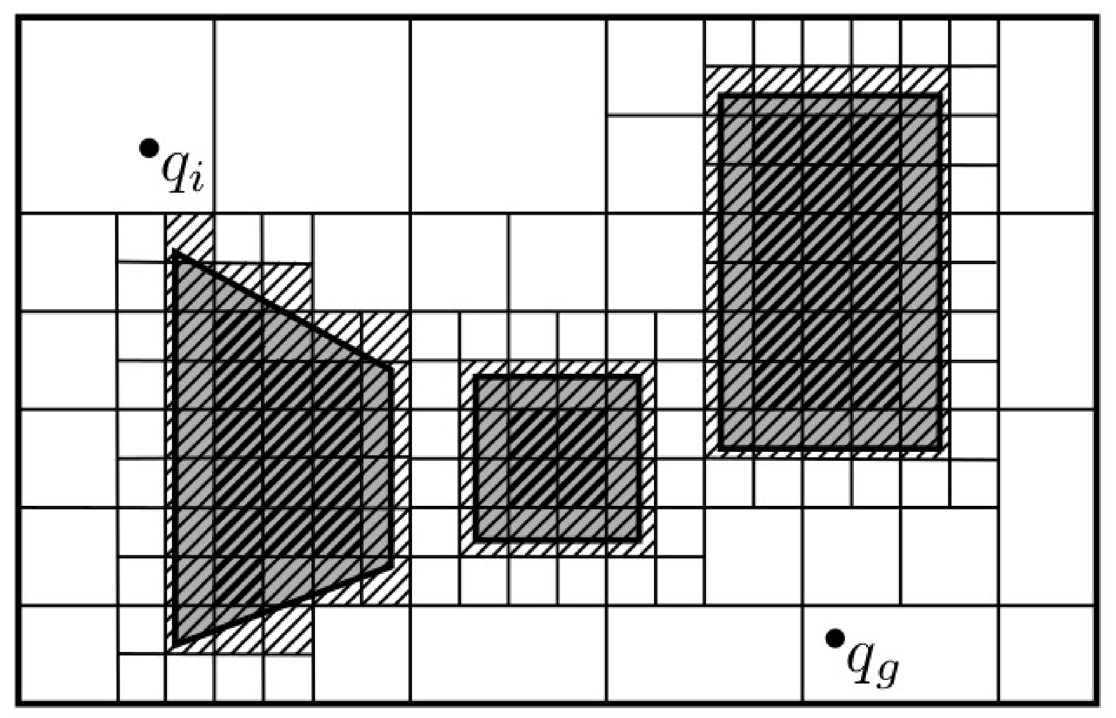
Blöchliger et al. (2017). Topomap: Topological Mapping and Navigation Based on Visual SLAM Maps. CoRR, http://arxiv.org/abs/1709.05533

# Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
- Introduction to assignment

# From map to graph

- Nodes
- Edges

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# From map to graph

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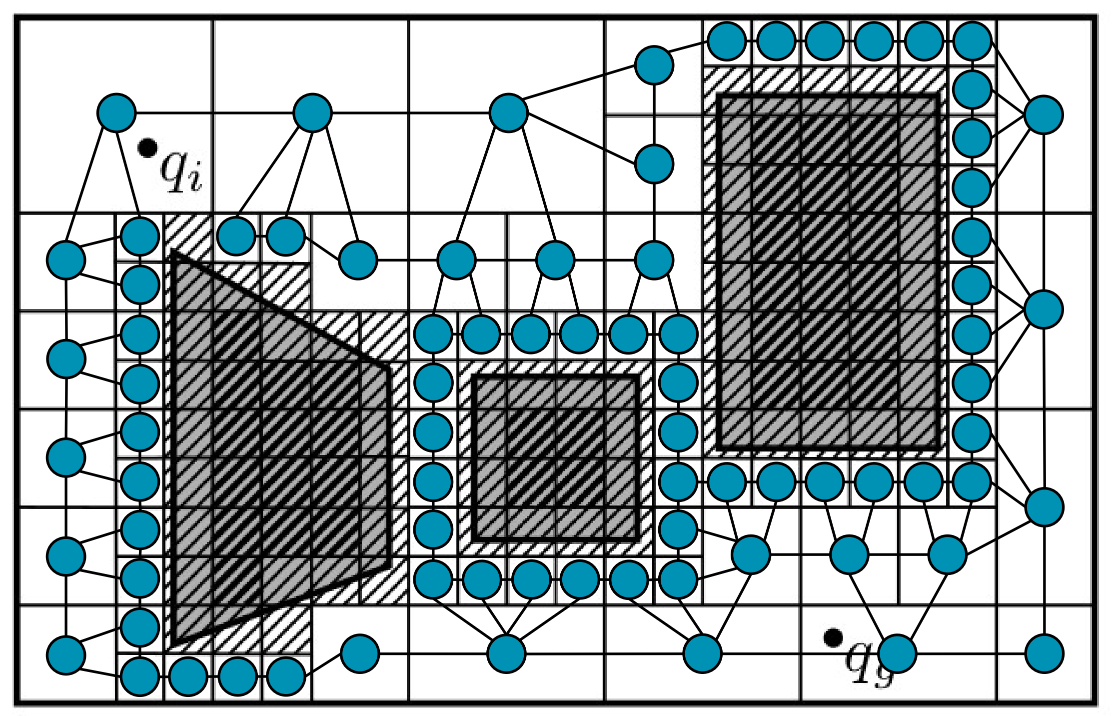


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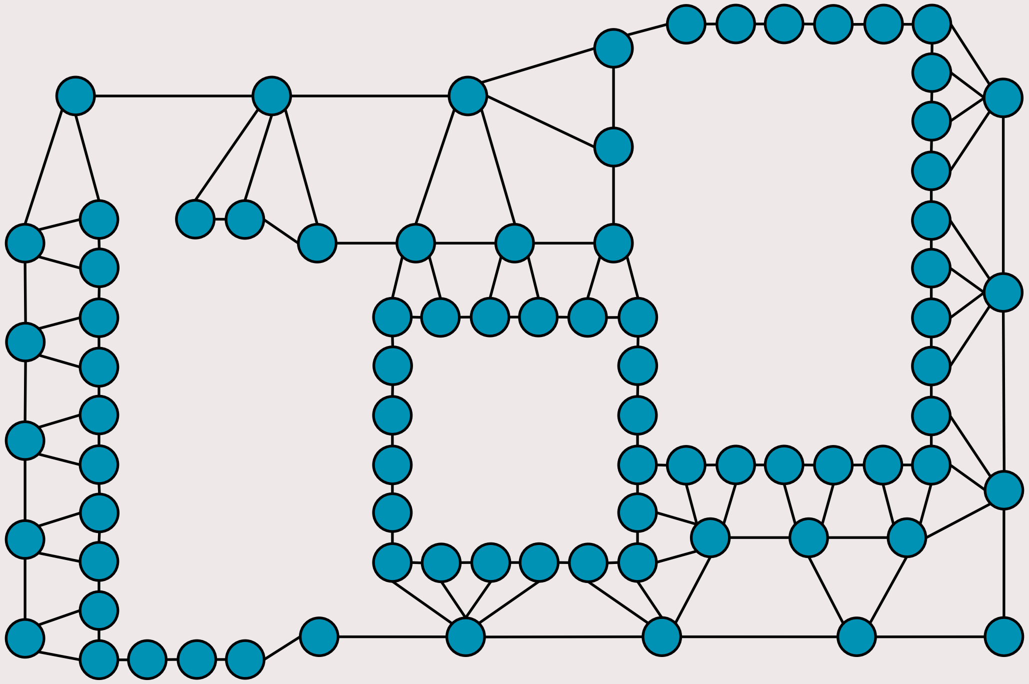
# From map to graph

- Nodes
- Edges



# From map to graph

- Nodes
- Edges


Source: https://www.openstreetmap.org/

# From map to graph

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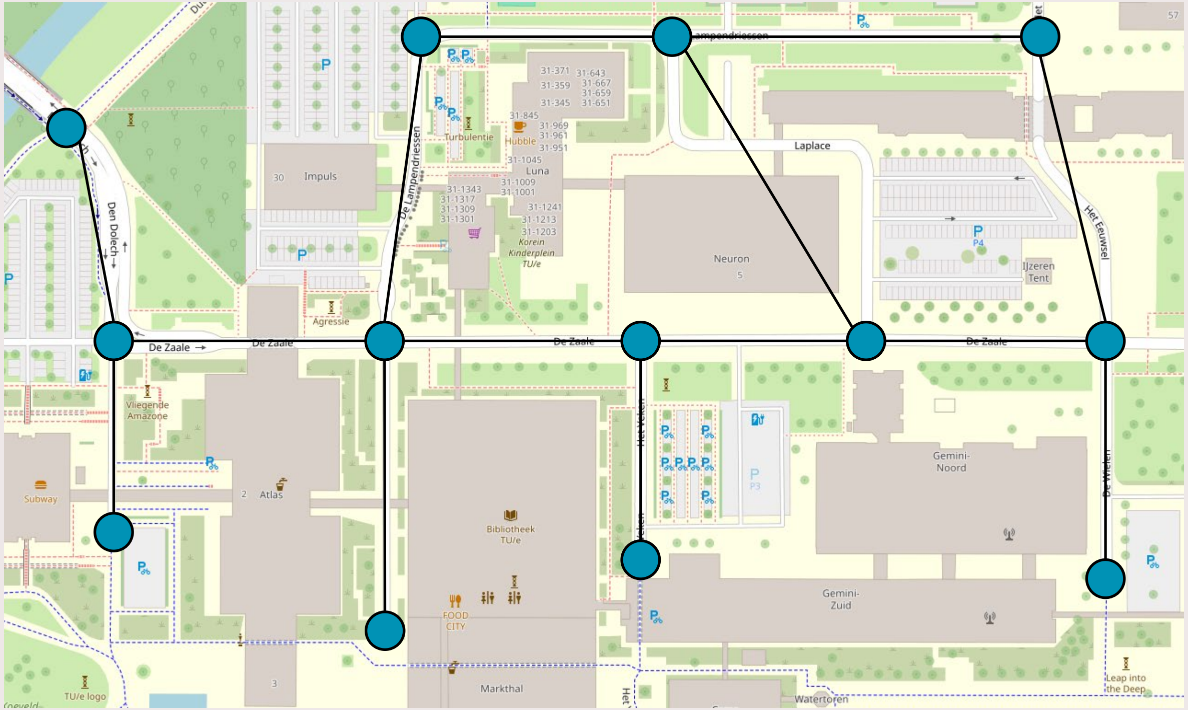


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# From map to graph

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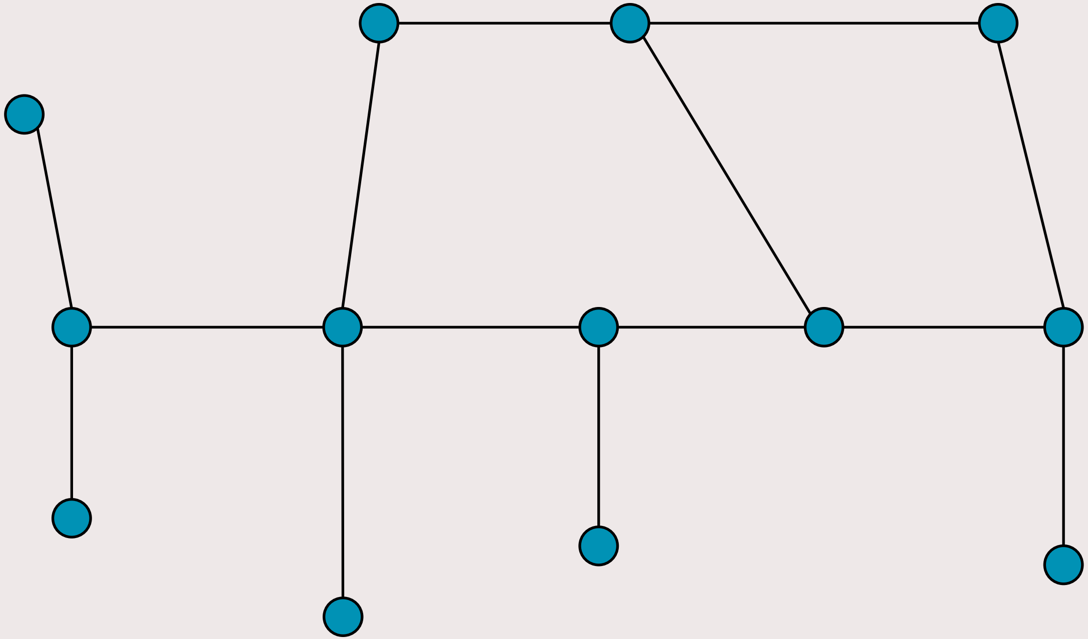

# Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
  - Dijkstra’s algorithm
  - A* algorithm
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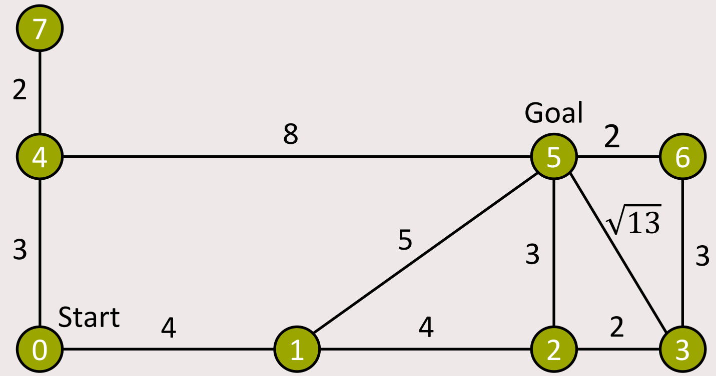
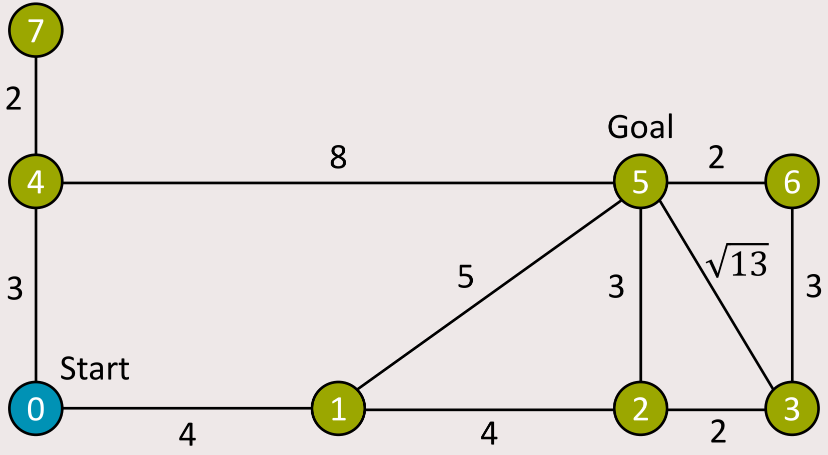
# Dijkstra's algorithm

- Goal: find the shortest path from start to goal in a graph
- Two stages:
  - Exploration starting from start node
  - Tracing back the path from goal to start
- Guarantees optimality!



# Dijkstra's algorithm / Exploration




| Step | Closed nodes $\{v \notin Q\}$ | Open nodes (distance) $\{v \in Q \mid \mathrm{dist}[v] < \infty\}$ | Unvisited nodes $\{v \in Q \mid \mathrm{dist}[v] = \infty\}$ |
|---|---|---|---|
| 0 | | 0 (0) | 1, 2, 3, 4, 5, 6, 7 |

```
function Dijkstra(Graph, start, goal):

for each node v in Graph.Nodes:
    dist[v] = INF
    prev[v] = NONE
    add v to Q
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while Q is not empty:
    u = node in Q with min dist[u]
    if u is goal:
        return dist, prev
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        d = dist[u] + Graph.Edges(u, v)
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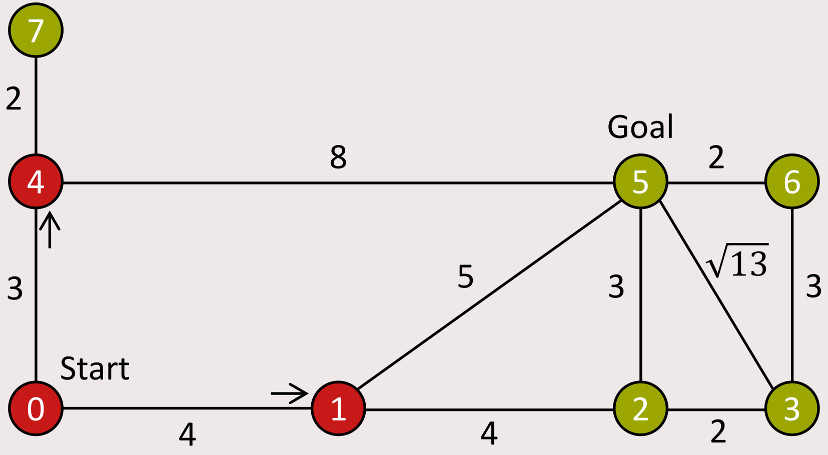
Pseudo-code based on https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

# Dijkstra's algorithm / Exploration

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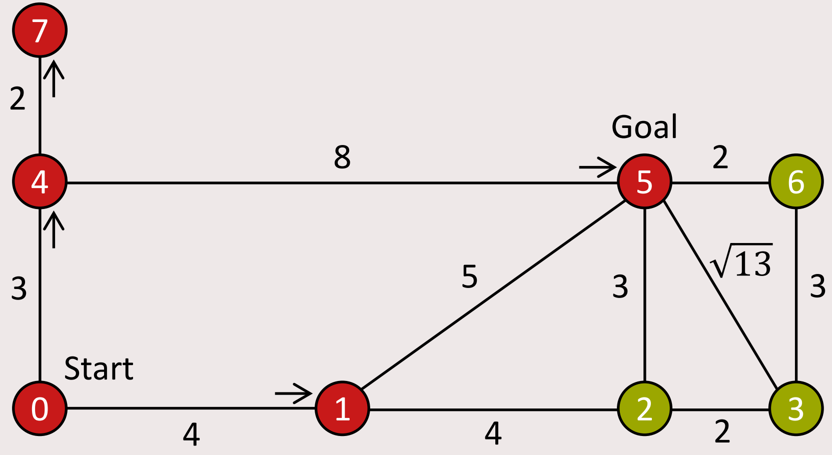


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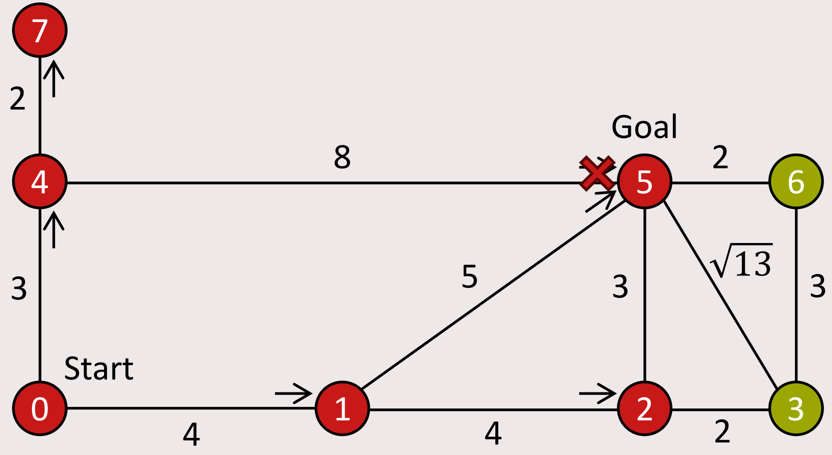


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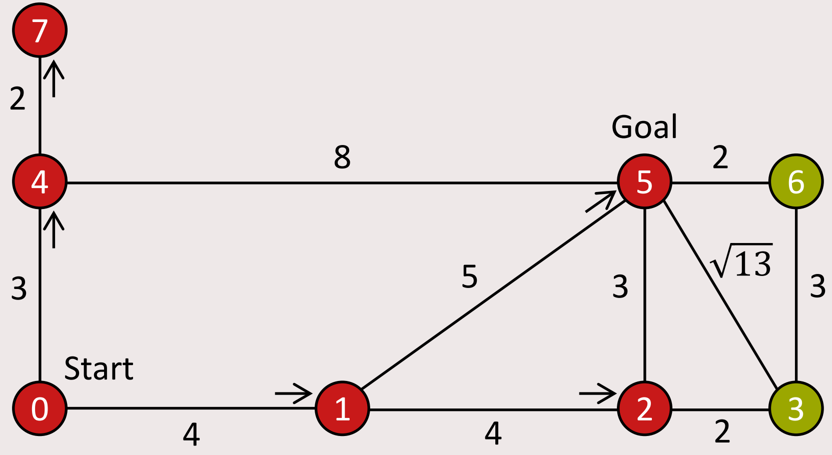


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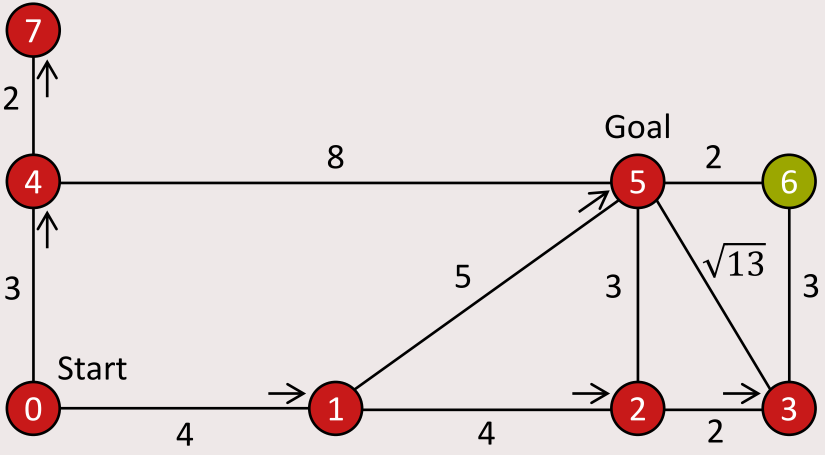


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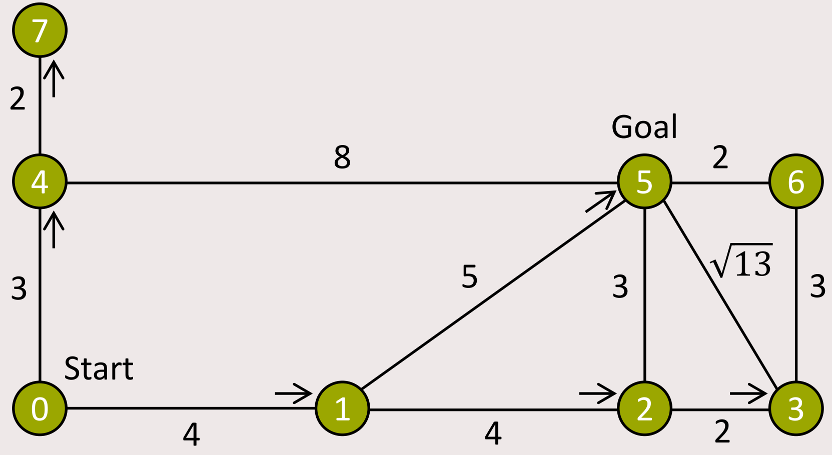
# **Dijkstra's algorithm** / Trace path back

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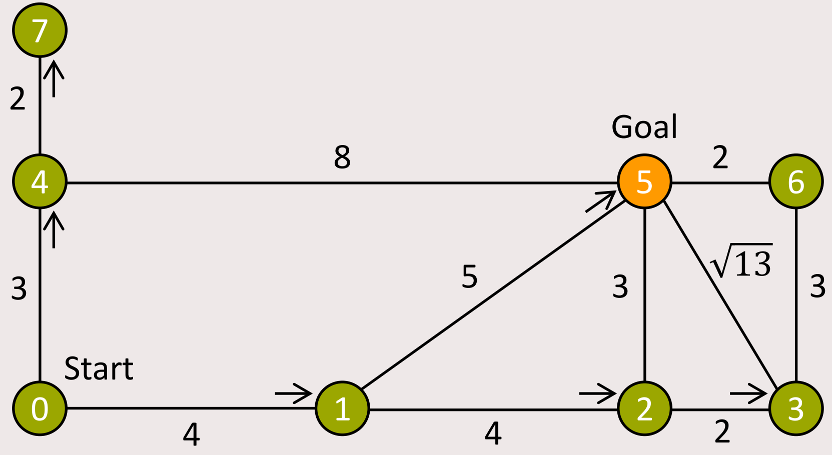


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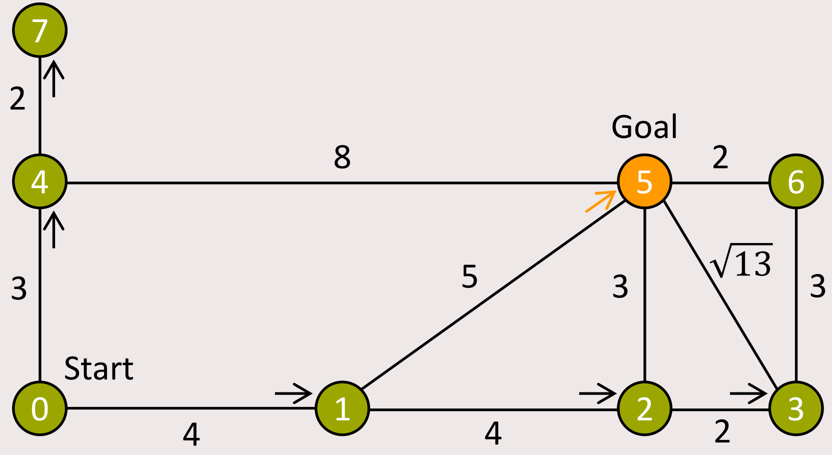


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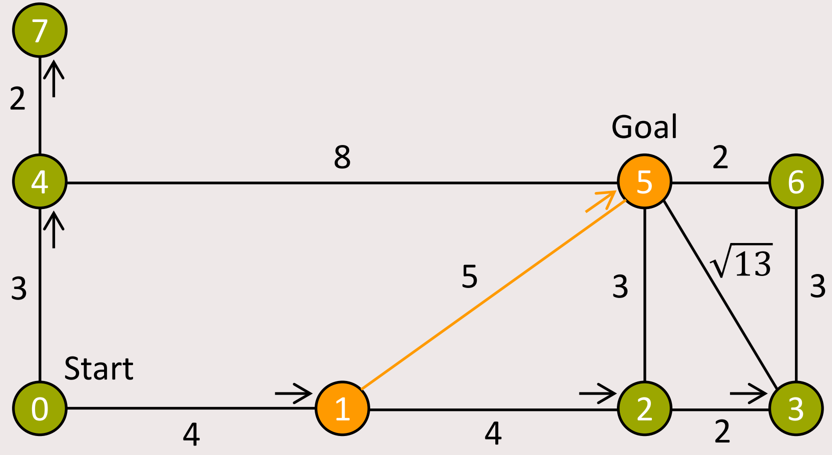


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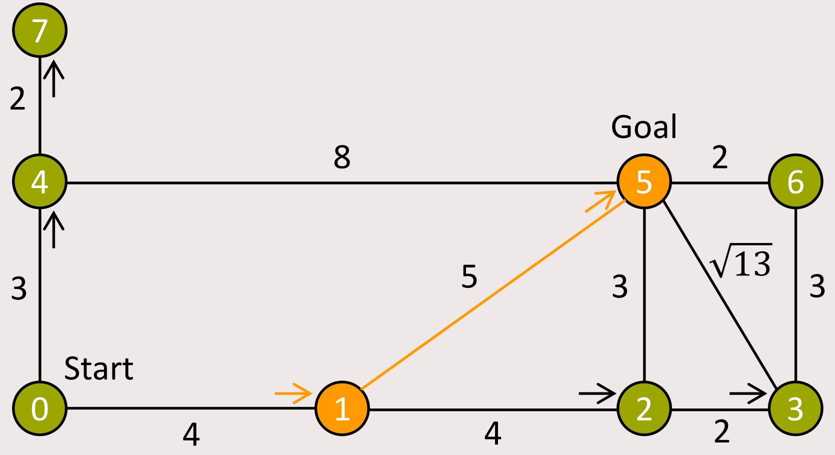


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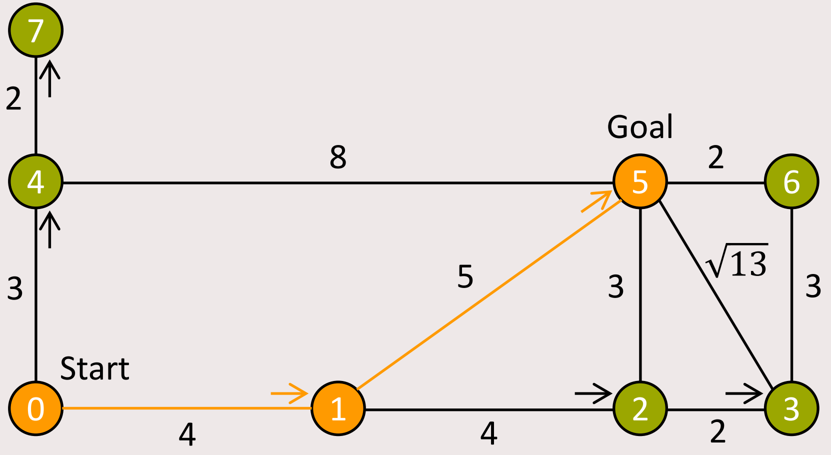


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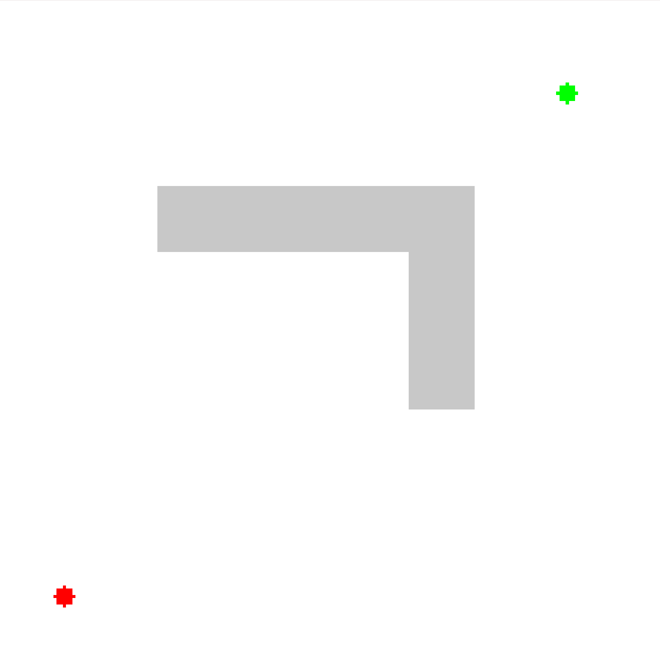
# **Dijkstra's algorithm** / Larger scale visualization



Dijkstra's algorithm. Source:
https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

# A* algorithm

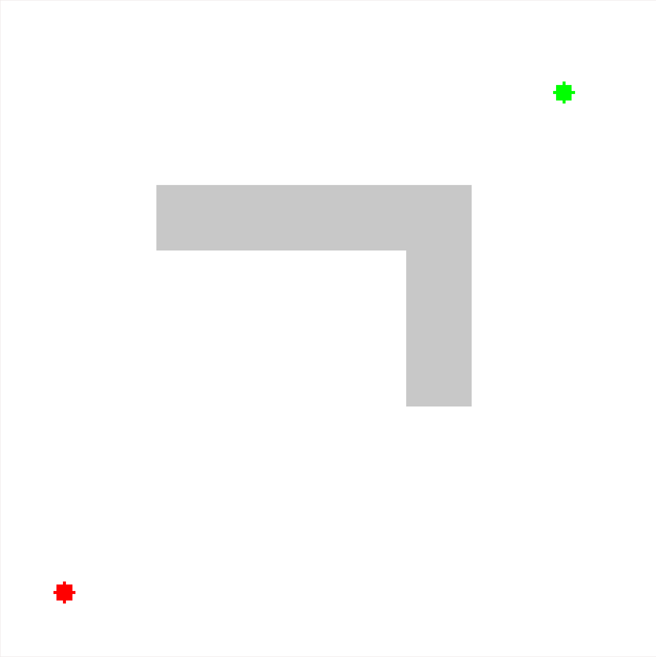
- "A* = Dijkstra + heuristic"
- Select open node with minimum:
  - Dijkstra: cost-to-come
  - A*: cost-to-come + heuristic cost-to-go
- Heuristic approximates remaining distance to goal
- Criteria for an admissible heuristic to guarantee optimality:
- Example: Euclidean distance to goal
  - $H(goal) = 0$
- Example: Euclidean distance to goal

```
function Astar(Graph, start, goal):

for each node v in Graph.Nodes:
    dist[v] = INF
    heur[v] = H(v)
    prev[v] = NONE
    add v to Q
    dist[start] = 0

while Q is not empty:
    u = node in Q with min dist[u] + heur[u]
    if u is goal:
        return dist, prev
    remove u from Q
    for each neighbor v of u still in Q:
        d = dist[u] + Graph.Edges(u, v)
        if d < dist[v]:
            dist[v] = d
            prev[v] = u
```

# A* algorithm / Visualization compared to Dijkstra



A* algorithm. Source:
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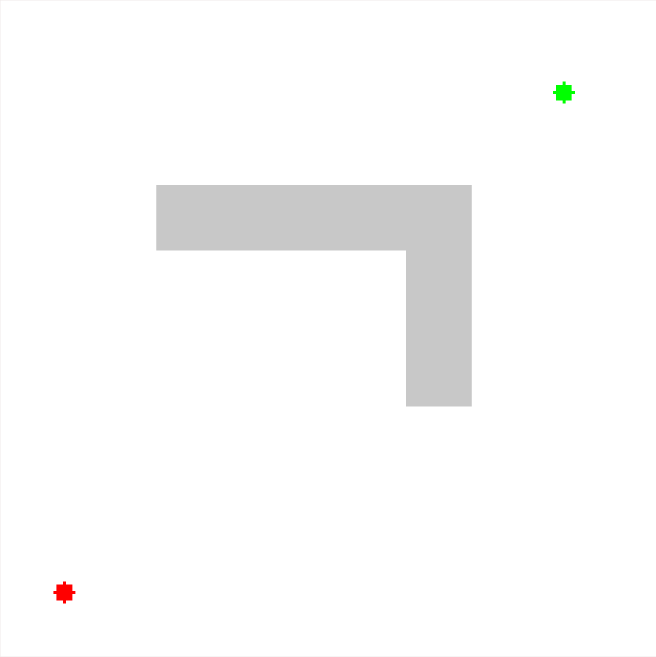

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# Break

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
- Introduction to assignment

Any questions so far?



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# Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
  - Visibility graphs
  - Rapidly-exploring Random Tree (RRT)
  - Probabilistic Roadmap
- Summary
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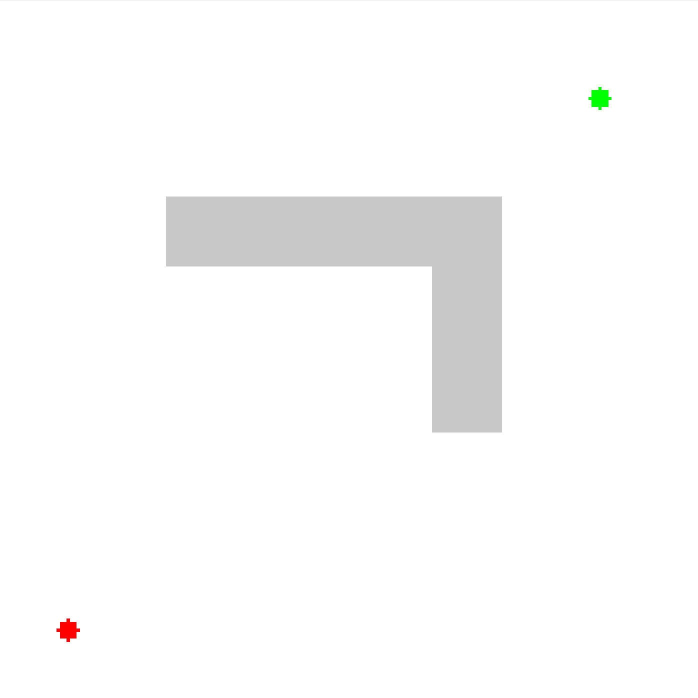
# Efficient graph generation / Motivation

- Example: A* on grid maps
  - Advantages:
    + No need to create the graph in advance, because the grid is regular
    + Easy to calculate path cost
  - Disadvantages:
    - In general less efficient because not all nodes are necessary



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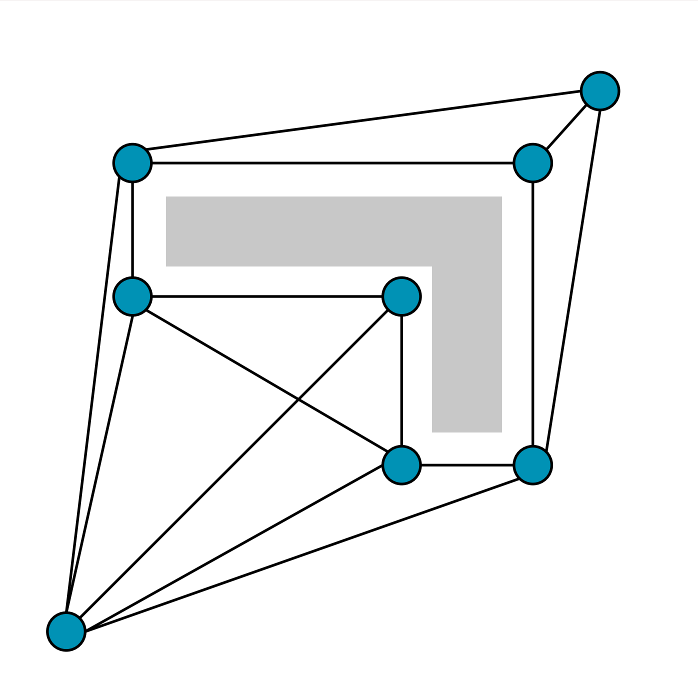
- More efficient graph desired!



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# **Efficient graph generation** / Visibility graph

- Nodes:
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  - Agent
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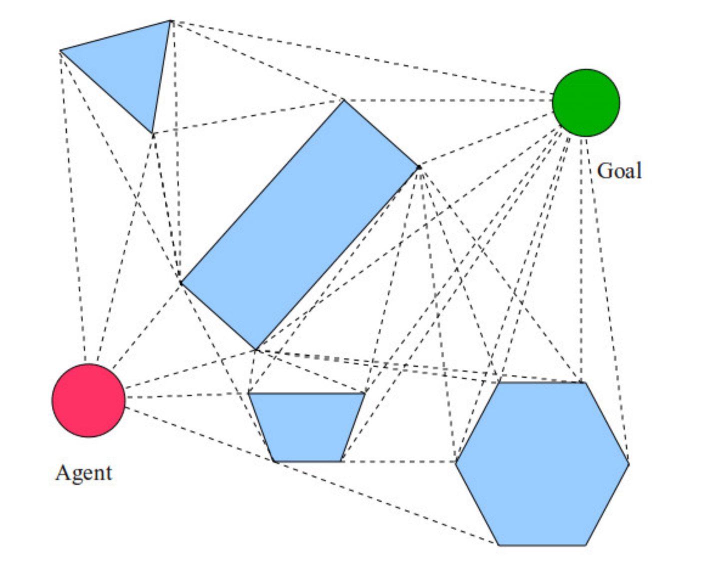
- Edges:
  - All straight lines between nodes that do not cross obstacles (visibility)

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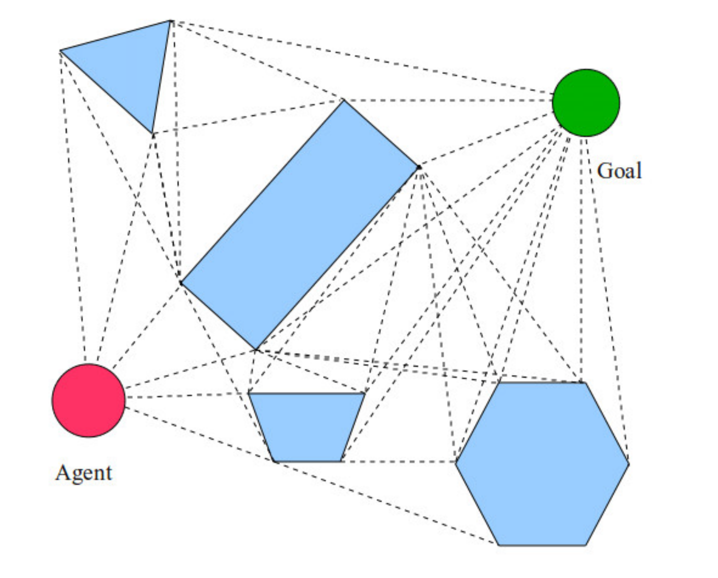


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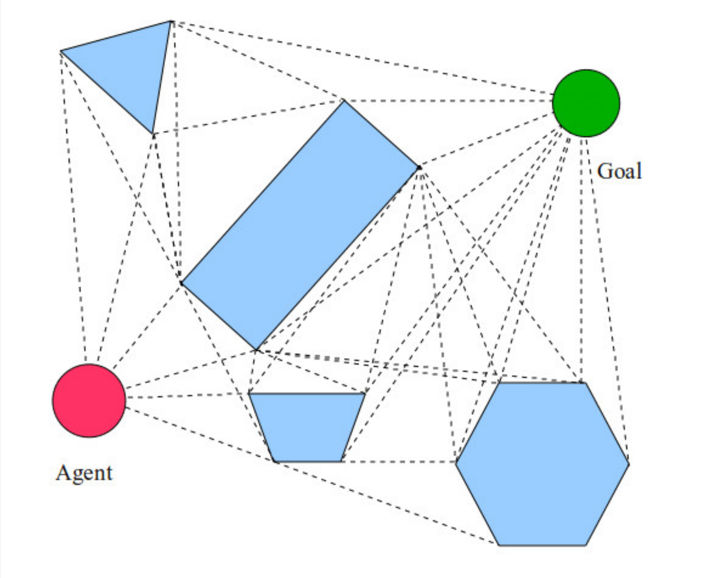
- Robustness: inflate obstacles or handled by local planner

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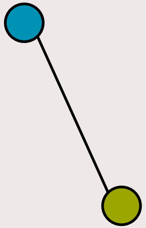


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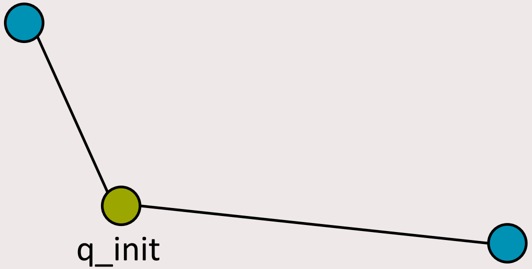


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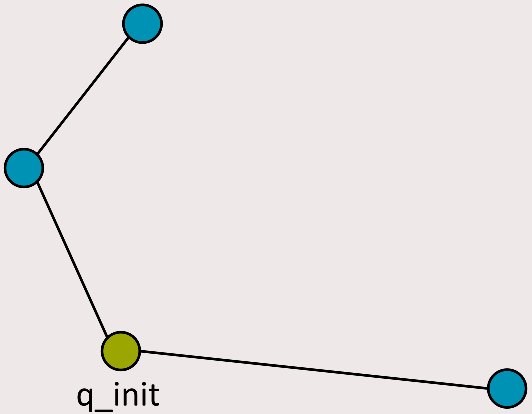


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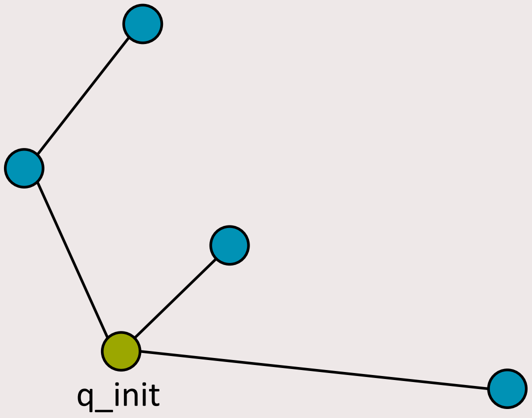


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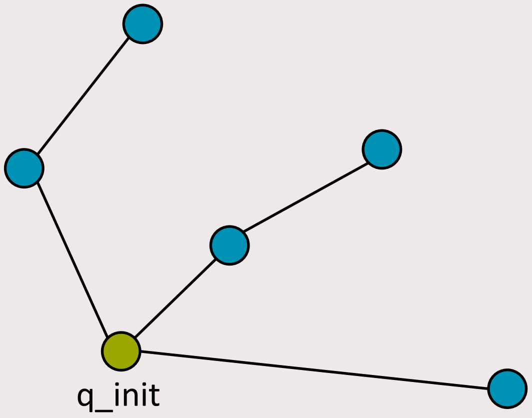


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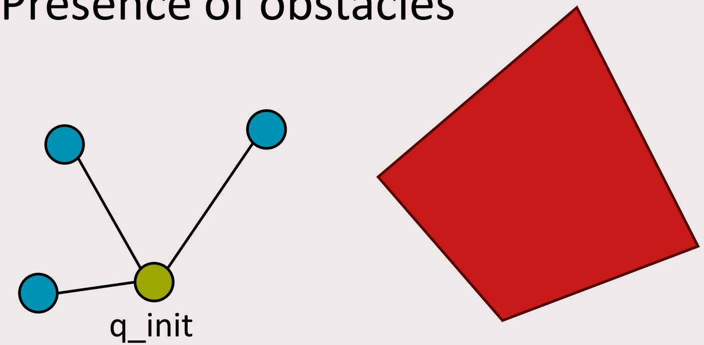
- RRT = Rapidly-exploring Random Tree
- Construct a tree by random sampling, starting at the initial state
- Tree is dense in the limit

# Efficient graph generation / RRT

- Presence of obstacles

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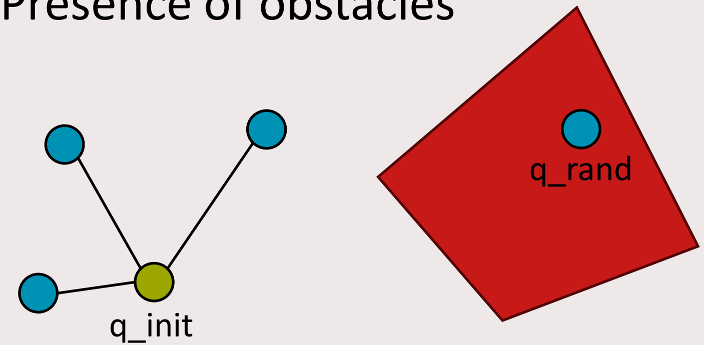


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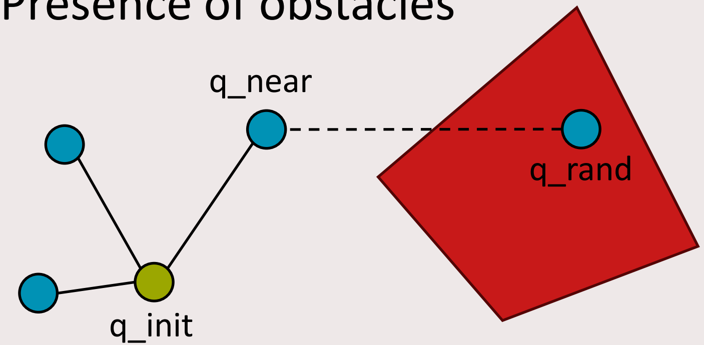


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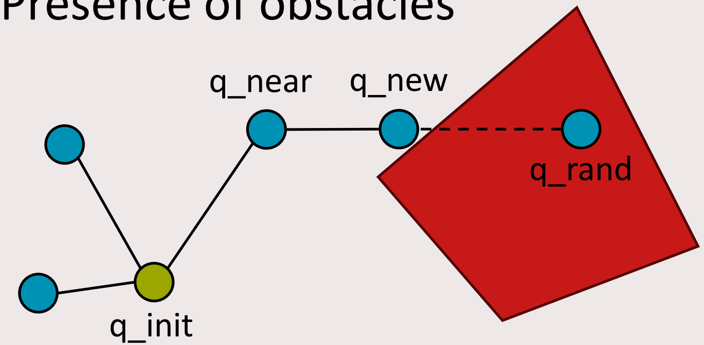


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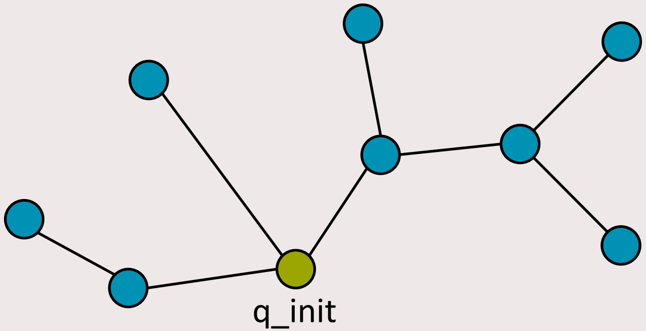
- RRT*: RRT with rewiring for shorter paths
  - Similar to Dijkstra and A*



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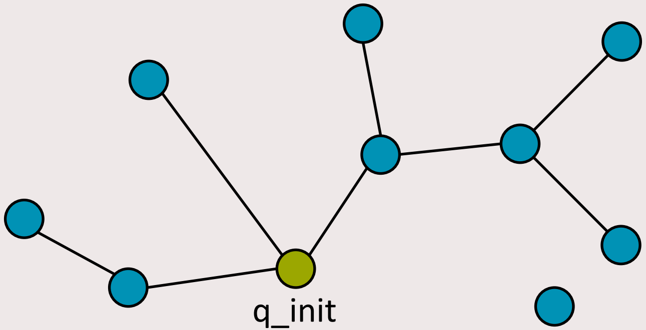

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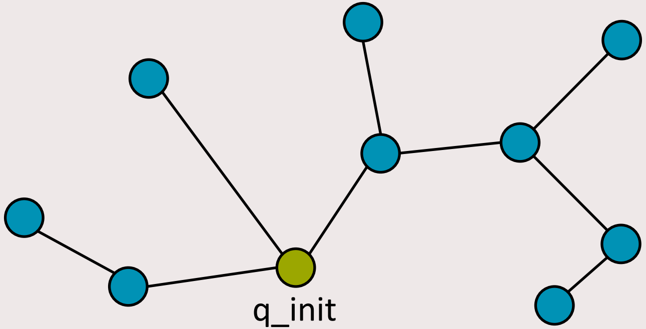

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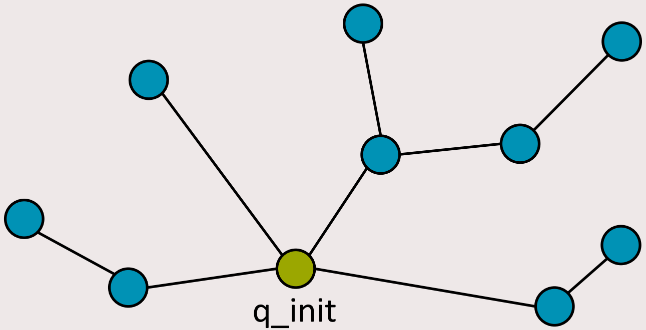
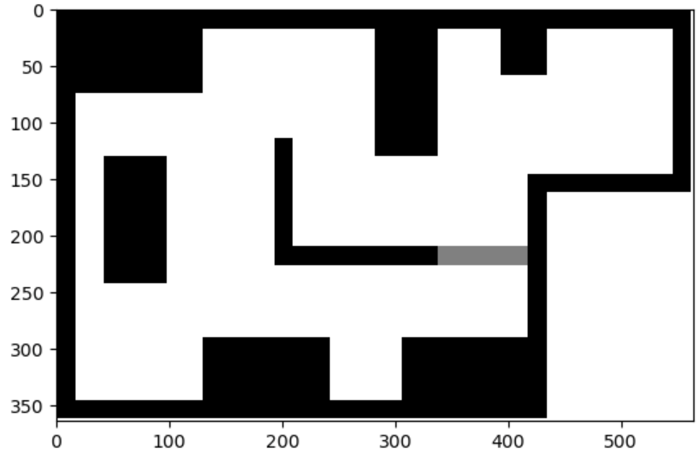


# Efficient graph generation / Probabilistic roadmap

- Nodes: randomly generated (valid) configurations
- Edges: (straight-line) collision-free connections between nodes



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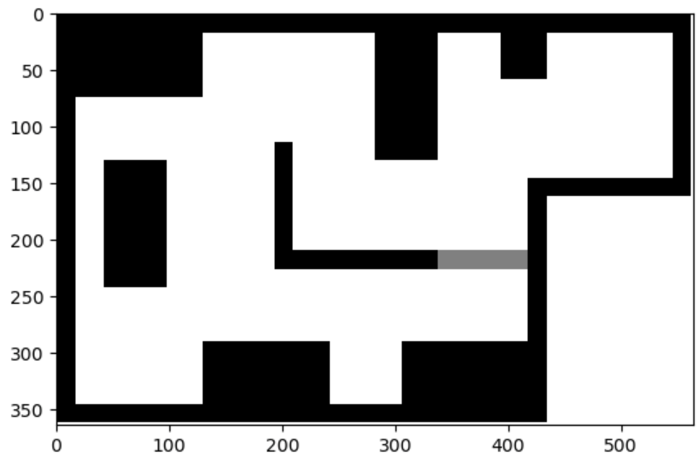

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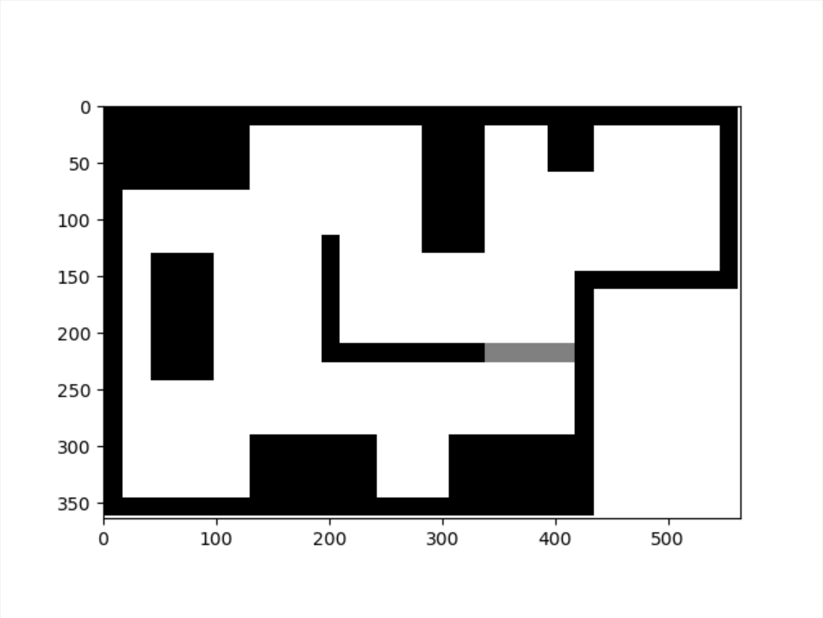
- Roadmap can be independent of start and final configurations



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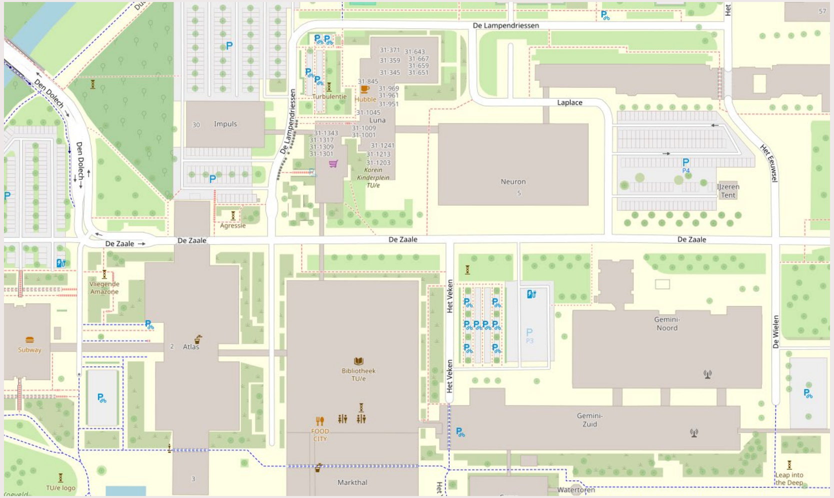
- Exact trajectory planning is often infeasible
- Would lead to constant replanning
- Local planner needed (previous lecture)
  - Example: carrot planner

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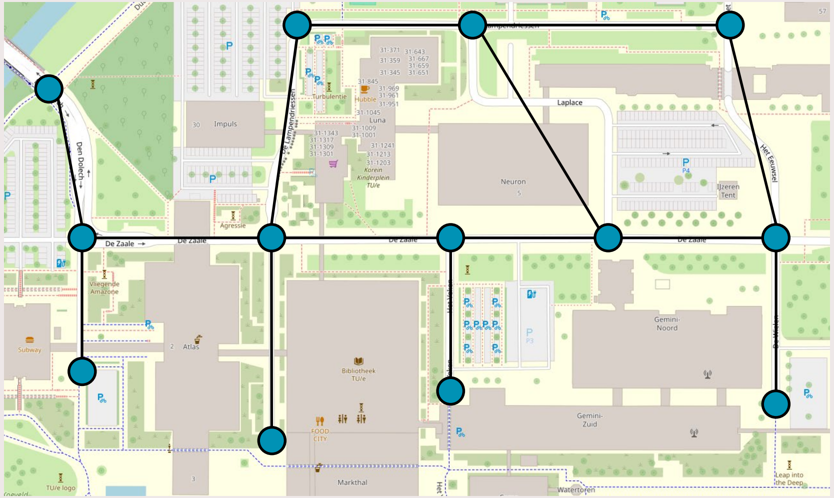


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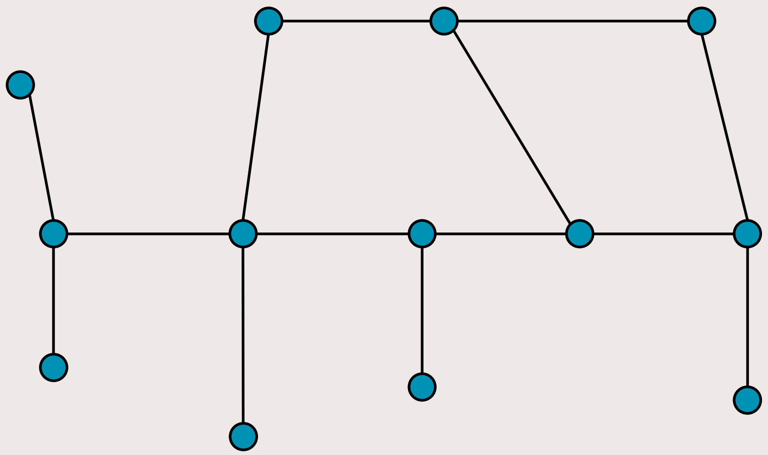

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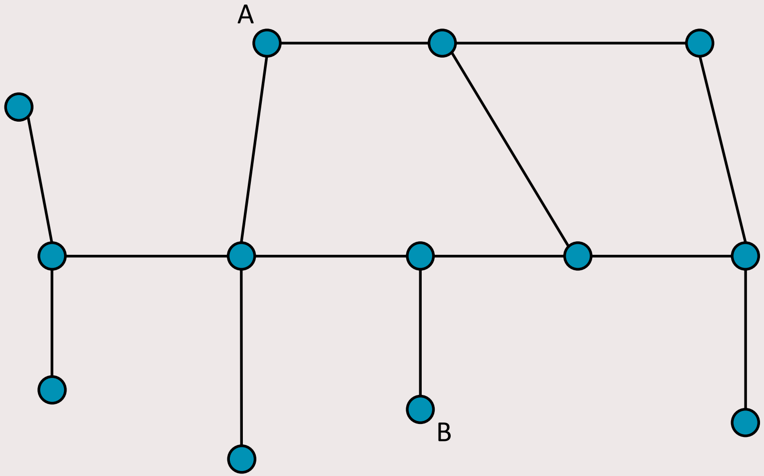

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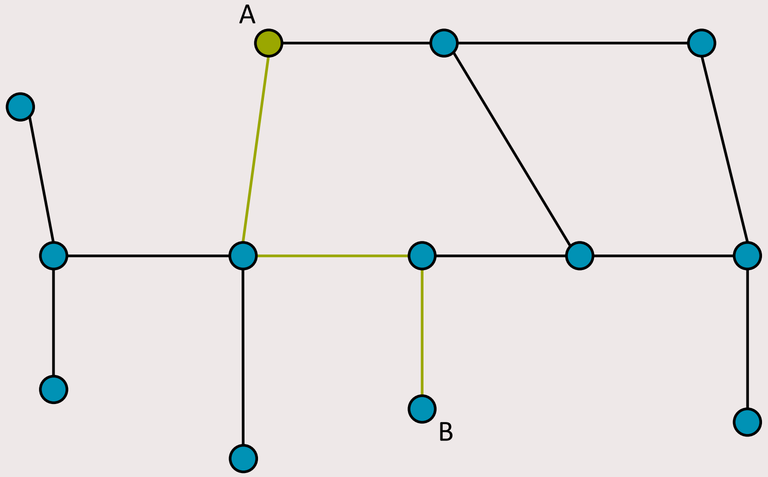

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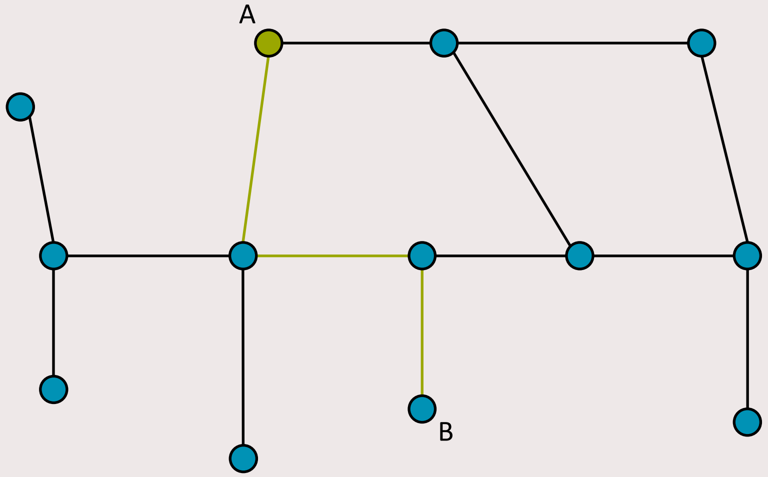

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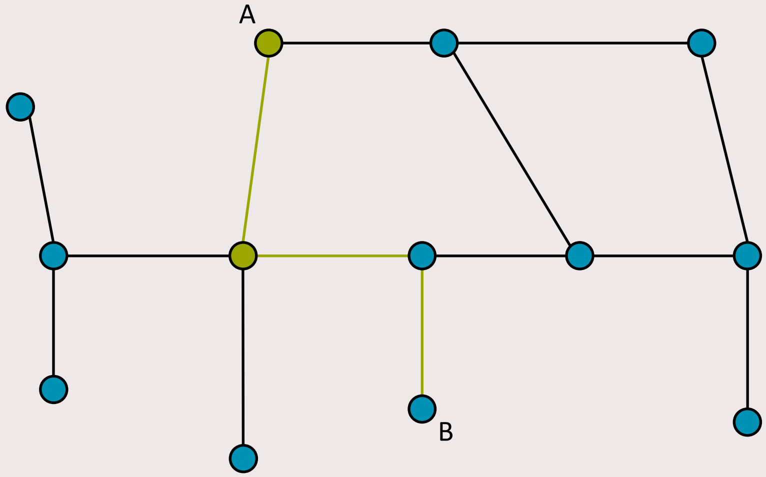

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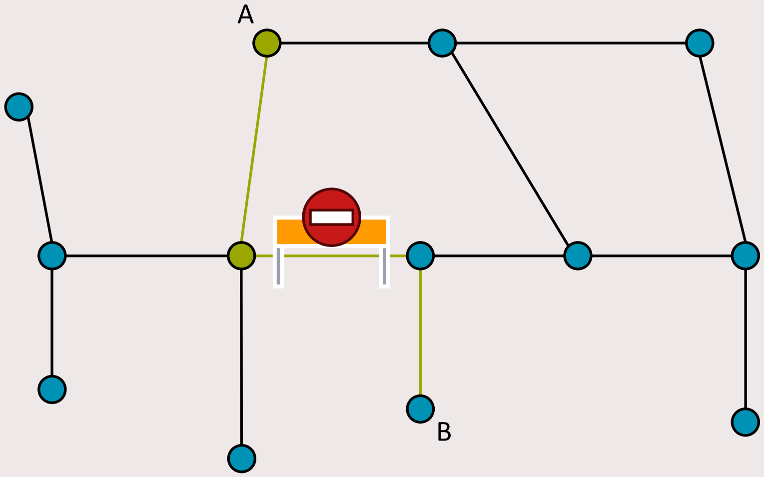

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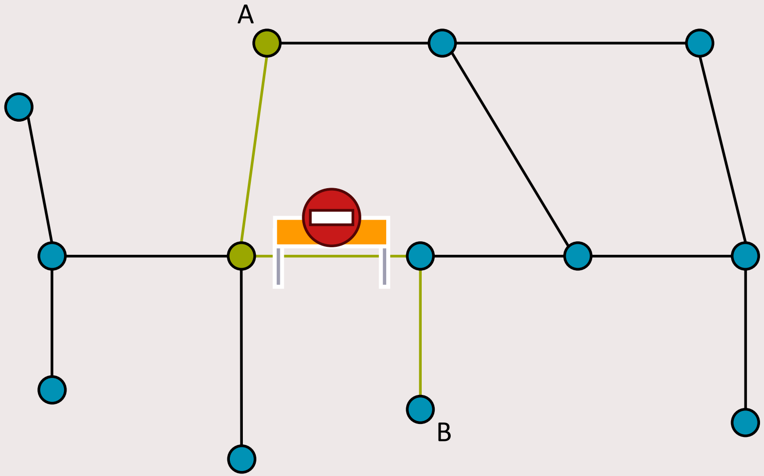

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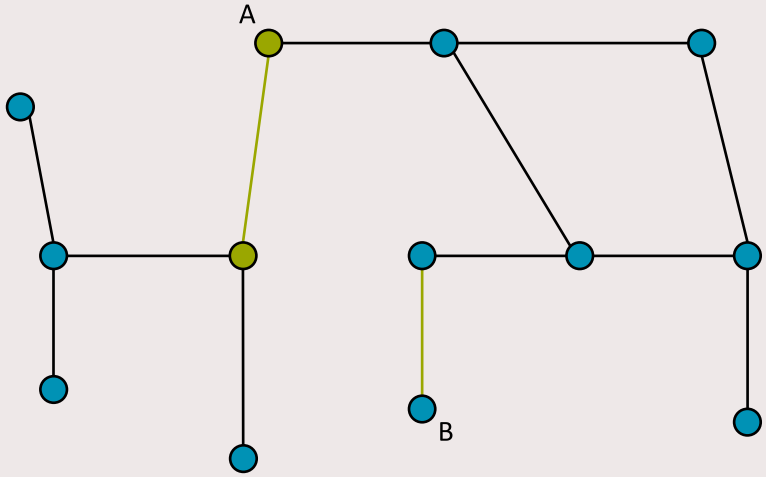

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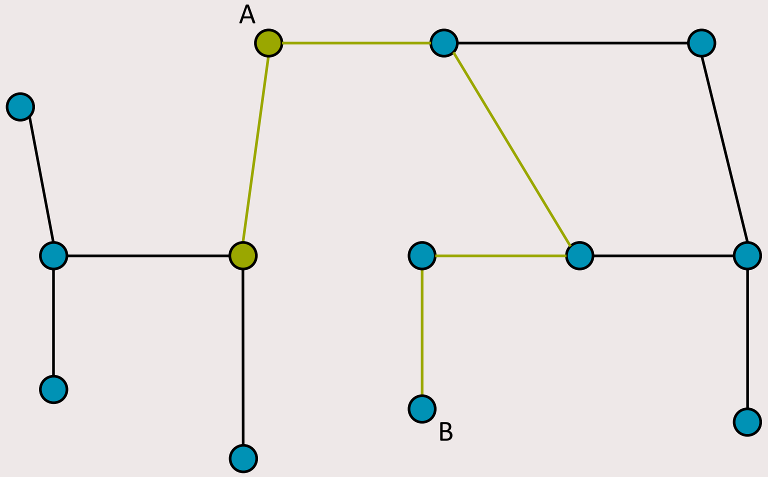

# Outline

- Recap local navigation & intro global navigation
- Map representations
- From map to graph
- Path planning in graphs
- Efficient graph generation
- Summary
- Introduction to assignment

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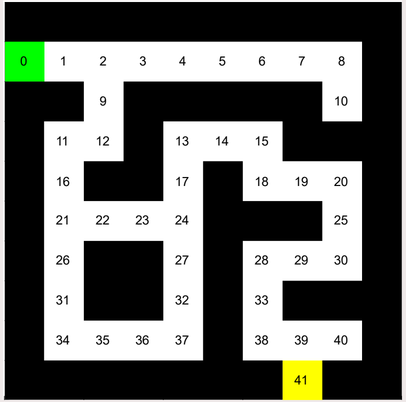
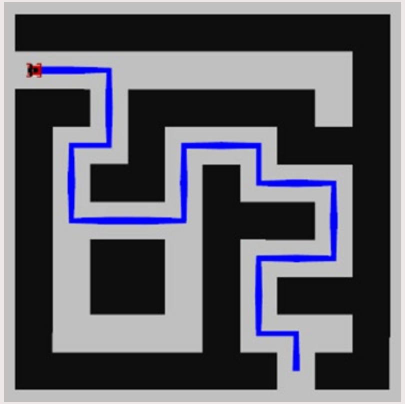
# Assignment part 1

- Complete an implementation of the A* algorithm to find the shortest path from start to finish in a maze
- Provided: list of nodes and edges, index of start and finish nodes
- Required: sequence of node indices that form the shortest path from start to finish



# Assignment part 2

- Complete the implementation of generating a graph that represents a Probabilistic Roadmap (PRM)
- When part 1 of the assignment is also finished, a path from start to goal in this PRM can be found using your A* algorithm

# Assignment part 3

- Connect global and local planner