Frequency response measurement in closed loop: brushing up our knowledge

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Abstract

Determination of the frequency response function of a system, using cross- and powerspectrum is a commonly used technique. In the closed loop situation, usually the sensitivity will be measured. The reason for doing this is sometimes a bit slipped away. Therefore this report will brush up our knowledge and consider the theoretical background.

First, a theoretical examination of the open loop case is done. Correlations are determined from the time-domain formulas. Using the fourier transform, they will be converted to spectra. The frequency response function can be determined in this way. This is also done by using the frequency domain approach from the start. The frequency domain approach is also used for the determination of the closed loop system. For the closed loop system, a number of methods to determine the system are found. These methods all have their own specific properties and are useful in different conditions.

The theoretical methods are tested in practice. The found properties are checked and other phenomena are examined.

Contents

Chapter 1

Introduction

The frequency response technique is often used for the identification of a system. For measuring a closed loop system different methods are available. Normally the sensitivity is measured. These are routines which are so common, they can be executed without realizing the theoretical background.

The goal of this report is to give more insight in the different methods and their theoretical background. In this way, they can be used in a well founded way.

To achieve this goal, a frequently used technique ([4] [1] [6]) will be re-investigated. The theoretical background of each method will be examined by writing out the system equation in the time and frequency domain for the different methods. The different methods will be judged for different situations. These methods are executed in a real situation to check the theoretical results. Practical difficulties that will occur during a measurement will be discussed.

Chapter 2

Theory for FRF measurement

The usual way to perform a frequency response function (FRF) measurement is to make use of cross- and autopowerspectra. In the open-loop case the measured crosspowerspectrum is divided by the measured autopowerspectrum to create an estimator for the frequency response function. The use of spectra takes care that uncorrelated extraneous noise does not disturb the estimated frequency response function.

When this analysis is done in a closed-loop environment, it is no longer usual to look at the input and the output. The above method will now give a biased result in the case of extraneous noise. The best method is to perform an analysis with the help of the sensitivity.

In this chapter a theoretical survey will be given to examine the differences and the coming about of the different methods.

2.1 Open loop theory

2.1.1 Derivation in the time domain

In this subsection, a derivation from the basic open-loop equation will be made.

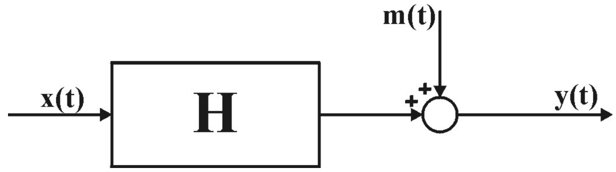

Figure 2.1: Scheme of open-loop system with additional noise

Here is taken a lineair open-loop system with input $x(t)$ and output $y(t)$. There is a source of extraneous noise $m(t)$ at the output which is assumed to be uncorrelated with $x(t)$. The system can be described in the timedomain with the next equation:

$$y(t) = h(t) \otimes x(t) + m(t) = \int_0^{+\infty} h(\eta)x(t-\eta)d\eta + m(t) \tag{2.1}$$

The time shifted product between the input and the output is:

$$x(t)y(t+\tau) = \int_0^{+\infty} h(\eta)x(t)x(t+\tau-\eta)d\eta + x(t)m(t+\tau) \tag{2.2}$$

The expectation of the time shifted product between two variables is defined as their correlation [1]:

$$\begin{aligned} R_{xy}(\tau) &= E\Big(x(t)y(t+\tau)\Big) \\ &= \tfrac{1}{T}\int_0^T x(t)y(t+\tau)dt \\ &= \tfrac{1}{T}\int_0^T \Big[\int_0^{+\infty} h(\eta)x(t)x(t+\tau-\eta)d\eta + x(t)m(t+\tau)\Big]dt \\ &= \tfrac{1}{T}\int_0^T \int_0^{+\infty} h(\eta)x(t)x(t+\tau-\eta)d\eta dt + \tfrac{1}{T}\int_0^T x(t)m(t+\tau)dt \\ &= \int_0^{+\infty} h(\eta)R_{xx}(\tau-\eta)d\eta + R_{xm}(\tau) \end{aligned} \tag{2.3}$$

The signals x(t) and m(t) are not correlated so: $R_{xm}(\tau) = 0$.

The definition of powerspectrum is the fourier transform of the correlation [1]. Applying the fourier transform on (2.3) gives:

$$\begin{aligned} S_{xy}(f) &= \int_{-\infty}^{+\infty} R_{xy}(\tau)e^{-2\pi jf\tau}d\tau \\ &= \int_{-\infty}^{+\infty}\int_{0}^{+\infty} h(\eta)R_{xx}(\tau-\eta)e^{-2\pi jf\tau}d\eta d\tau \\ &= \int_{0}^{+\infty} h(\eta)e^{-2\pi jf\eta}d\eta \int_{-\infty}^{+\infty} R_{xx}(t)e^{-2\pi jft}dt \quad (With: \quad t=\tau-\eta; \quad dt=d\tau) \\ &= H(f)S_{xx}(f) \end{aligned} \tag{2.4}$$

This is a very useful result. It means that there can be made an estimation of the FRF which does not depend on $m(t)$ as long as it is not correlated with $x(t)$.

$$\hat{H}_1(f) = \frac{S_{xy}}{S_{xx}} \tag{2.5}$$

An estimator which is also used in literature [3] can be derived in the same way. In the areas near resonance peaks where large gains apply, this estimator gives a better accuracy while the first estimator gives a better accuracy at lower gains.

$$\hat{H}_2(f) = \frac{S_{yy}}{S_{yx}} \tag{2.6}$$

An other result that can be found in the same way:

$$S_{yy}(f) = |H(f)|^2 S_{xx}(f) + Snn(f) \tag{2.7}$$

The coherence, which tells something about the relation of the energy at the input and the output is defined as follows: [1]:

$$\gamma_{xy}^2(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)} \tag{2.8}$$

Written out for this specific case it gives:

$$\gamma_{xy}^2(f) = \frac{|H(f)|^2 S_{xx}(f)^2}{|H(f)|^2 S_{xx}(f)^2 + S_{xx}(f)S_{nn}(f)} \tag{2.9}$$

This makes γ^2 a good estimator for the quantity of noise on the output and the quality of the measurement.

In literature [2] also can be found:

$$var[\hat{H}_{xy}(f)] = \frac{(1-\gamma_{xy}^2(f))|\hat{H}_{xy}(f)|}{2\gamma_{xy}^2(f)N} \tag{2.10}$$

A time domain analysis for the closed loop system can be found in appendix A

2.1.2 Derivation in the frequency domain

Spectral density functions can also be found via fourier transforms. This gives an important and handy tool for further analysis. [2]

$$S_{xy}(f) = \lim_{T\to\infty} \frac{1}{T} X^*(f)Y(f) \tag{2.11}$$

where X^* is the complex conjugated of X and T the total sample time.

The same procedure can be followed in the frequency domain. Now the system can be described as:

$$Y(f) = H(f)X(f) + M(f) \tag{2.12}$$

Combining the above, similar relation like equation (2.4) can be found in an easy way:

$$\begin{aligned} S_{xy}(f) &= \tfrac{1}{T}\Big(X^*(f)[H(f)X(f) + M(f)]\Big) \\ &= \tfrac{1}{T}\Big(H(f)X^*(f)X(f) + X^*(f)M(f)\Big) \\ &= H(f)S_{xx}(f) + S_{xm}(f) \end{aligned} \tag{2.13}$$

Where again $S_{xm}(f) = 0$. Now the same result as in (2.6) is derived.

$$\hat{H}_1(f) = \frac{S_{xy}}{S_{xx}} \tag{2.14}$$

2.2 Closed loop theory

Many real processes become unstable or out of specifications without controller. Because these reasons, a lot of processes can't be analyzed in open loop. Now a frequency domain approach like in section (2.1.2) will be followed for the closed loop system. In each subsection a method will be discussed which can directly be implemented for the measurement of a closed loop system.

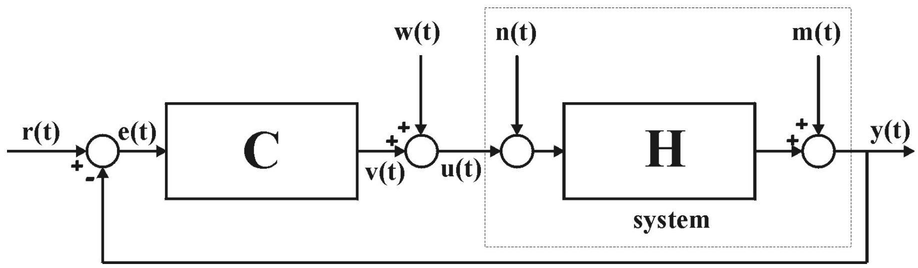

Figure 2.2: Scheme of the system

Here we are dealing with a lineair system in closed loop. The input $r(t)$ describes a reference trajectory of reference point. Both $n(t)$ and $m(t)$ are random uncorrelated noises which can not be influenced by any input. For the analysis we need the system to be excitated. Therefore we will use an extra input $w(t)$.

The system equations for different points in the loop are written down below.

$$\begin{aligned}
U(f) &= S(f)C(f)R(f) + S(f)W(f) - S(f)C(f)H(f)N(f) - S(f)C(f)M(f) \\
Y(f) &= S(f)C(f)H(f)R(f) + S(f)H(f)W(f) + S(f)H(f)N(f) + S(f)M(f) \\
V(f) &= S(f)C(f)R(f) - S(f)C(f)H(f)W(f) - S(f)C(f)H(f)N(f) - S(f)C(f)M(f) \\
E(f) &= S(f)R(f) - S(f)H(f)W(f) - S(f)H(f)N(f) - S(f)M(f)
\end{aligned} \tag{2.15}$$

Where

$$S(f) = \frac{1}{1 + C(f)H(f)} \tag{2.16}$$

For the closed loop case equation (2.11) will also be used.

$$S_{xy}(f) = \lim_{T\to\infty} \frac{1}{T} X^*(f)Y(f) \tag{2.17}$$

From now on we assume large time samples and leave the limit sign away. Again, there is assumed the different inputs are independent, so the correlations and spectra between them are zero.

$$S_{rw} = S_{rn} = S_{rm} = S_{wn} = ... = 0 \tag{2.18}$$

2.2.1 Direct method (y/u)

The direct method is the direct measurement of the system. Like the open-loop case the output of the system will be divided by the input.

$$\begin{aligned} S_{uy} &= \tfrac{1}{T}U^*Y = \tfrac{1}{T}\Big(S^*SC^*CHS_{rr} + S^*SHS_{ww} - S^*SC^*H^*HS_{nn} - S^*SC^*S_{mm}\Big) \\ S_{uu} &= \tfrac{1}{T}U^*U = \tfrac{1}{T}\Big(S^*SC^*CS_{rr} + S^*SS_{ww} + S^*SC^*CH^*HS_{nn} + S^*SC^*CS_{mm}\Big) \end{aligned} \tag{2.19}$$

$$\hat{H} = \frac{S_{uy}}{S_{uu}} = \frac{C^*CHS_{rr} + HS_{ww} - C^*H^*HS_{nn} - C^*S_{mm}}{C^*CS_{rr} + S_{ww} + C^*CH^*HS_{nn} + C^*CS_{mm}} \tag{2.20}$$

When only the signal S_{ww} is present, the FRF, $H(f)$ is measured. However, when there is extraneous noise, this takes unlike the open-loop case care for a biased estimate of the FRF. In the extreme situation with only noise $-1/C$ will be measured.

Writing out the whole coherence would make no sense because it would become very large and inconvenient. For the illustration only the terms w(t) and n(t) are taken along.

$$\gamma_{uy}^2 \approx \frac{|H|^2|S|^4S_{ww}^2 - 2|C||H|^3|S|^4S_{ww}S_{nn} + |C|^2|H|^4|S|^4S_{nn}^2}{|H|^2|S|^4S_{rr}^2 + 2|C|^2|H|^4|S|^4S_{ww}S_{nn} + |C|^2|H|^4|S|^4S_{nn}^2} \tag{2.21}$$

There can be concluded that by using the direct method, undesired noise causes a distortion in the estimated FRF. When there is only undesired noise, even -1/C is measured. The coherence function doesn't give much grip too because in the case of only extraneous noise it will also go to one.

2.2.2 Sensitivity (u/w)

The most common technique for measurements in closed loop is to determine the sensitivity.

$$S_{wu} = \frac{1}{T}W^*U = \frac{1}{T}SS_{ww} \tag{2.22}$$

$$\hat{S} = \frac{S_{wu}}{S_{ww}} = S \tag{2.23}$$

The advantage can directly been seen. When measuring the sensitivity, unlike the direct method there will be no bias.

The sensitivity and the FRF are defined as:

$$S(f) = \frac{1}{1 + C(f)H(f)} \tag{2.24}$$

$$H(f) = \frac{1 - S(f)}{C(f)S(f)} \tag{2.25}$$

The coherence is:

$$\gamma_{wu}^2 = \frac{S_{ww}}{|C|^2 S_{rr} + S_{ww} + |C|^2|H|^2 S_{nn} + |C|^2 S_{mm}} \tag{2.26}$$

The advantage of measuring the sensitivity is that undesired uncorrelated noise averages itself out. Another advantage is that the coherence will become lower with more outside noise, which makes it a good measurement-quality-indicator like in the open loop case.

A clear disadvantage is the determination of the FRF out of the sensitivity. First the controller has to be known or estimated. Second it takes an extra step which makes the interpretation of the coherence less clear.

2.2.3 Proces Sensitivity (y/w)

$$S_{wy} = \frac{1}{T} W^* Y = \frac{1}{T} SHS_{ww} \tag{2.27}$$

$$\hat{SH} = \frac{S_{wy}}{S_{ww}} = SH \tag{2.28}$$

Using the proces sensitivity, also no bias will appear due to extraneous noise.

The coherence:

$$\gamma_{wy}^2 = \frac{|H|^2 S_{ww}}{|C|^2|H|^2 S_{rr} + |H|^2 S_{ww} + |H|^2 S_{nn} + S_{mm}} \tag{2.29}$$

2.2.4 Combining Proces Sensitivity and Sensitivity

A direct estimate can be obtained bij dividing the Proces Sensitivity by the sensitivity.

$$SH = \frac{S_{wy}}{S_{ww}} \tag{2.30}$$

$$S = \frac{S_{wu}}{S_{ww}} \tag{2.31}$$

$$\hat{H} = \frac{SH}{S} = \frac{S_{wy}}{S_{wu}} \tag{2.32}$$

This gives a unbiased result like the sensitivity and the proces sensitivity. The pleasance of this method is that there is no knowledge of the controller necessary to determine the FRF.

Determining the coherence gives a problem because we are dealing with three different signals. A suggestion for a coherence look-a-like would be the multiplication of the coherence of the sensitivity and the proces sensitivity.

Chapter 3

A practical approach

When the theoretical formulas of chapter (2) are applied, in reality several problems will occur. In this chapter several of these problems are discussed, theoretically founded and solutions are carried out if possible.

3.1 Measured system and equipment

In this report several measurements will be discussed. These measurements are all carried out on a fourth-order system with two rotating masses, a spring and some damping. A schematic view of the system is represented below. The input u of the system is the applied voltage to the amplifier, this is assumed linear with the torsion T. The output is the rotation φ_1 or φ_2.

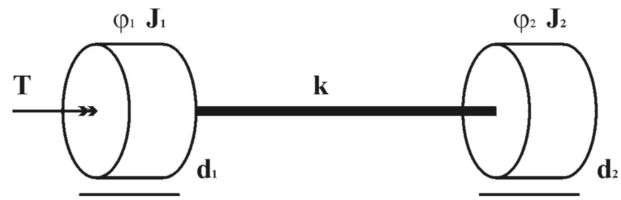

Figure 3.1: Schematic view of the model

For the data acquisition and the controller output the TUeDACS system has been used. This is a system which can be connected to a computer with a PCMCIA card. SIMULINK from MATLAB is used to create schemes and compile them. The sampling frequency during the experiments was 1000 Hz. The output signal to the amplifier is continuous and the signal is between minus 2.5 and plus 2.5 volts. The angle is measured by discrete encoderdisks with a accuracy of 2000 pulses per cycle.

After some experiments the parameters of the system have been determined. In this way, the experimental data can be compared with a theoretical FRF based on these parameters. With this knowledge two controllers have been made based on loopshaping. One for the controlling of the first mass, and the second for the controlling of the second mass. Only the FRF of the first mass will be further discussed.

3.2 Procedure

During the experiment the reference of the system describes a continuous rotational speed. The coulomb friction now works in one direction, which eliminates this non-linearity.

After executing the experiments, the data samples are detrended for the jogmode if so desired. This has been done by detrending a lineair fit. Next, the data samples are cut in a given number of pieces and possibly windowed. The fourier transform of each piece is taken and they are averaged to get a accurate result. This procedure is known as Welch Averaging Method [4]. Longer pieces of data samples give a better result for the lower frequencies, on the other hand give shorter pieces more averaging steps and a better result for the higher frequencies. Longer measurements give an overall improvement, but a time costly activity. A optimum has to be found, depending of the quantity of extraneous noise. When fourier transforms have been found the procedures described in section (2.2) can be followed. This is automatically done with the commando 'tfe' from MATLAB.

3.3 Sensitivity

An example of a good measurement using the sensitivity is showed below. With knowledge of the controller, the FRF is determined. The coherence of the sensitivity (not the system!) is plotted to say more about quality of the measurement.

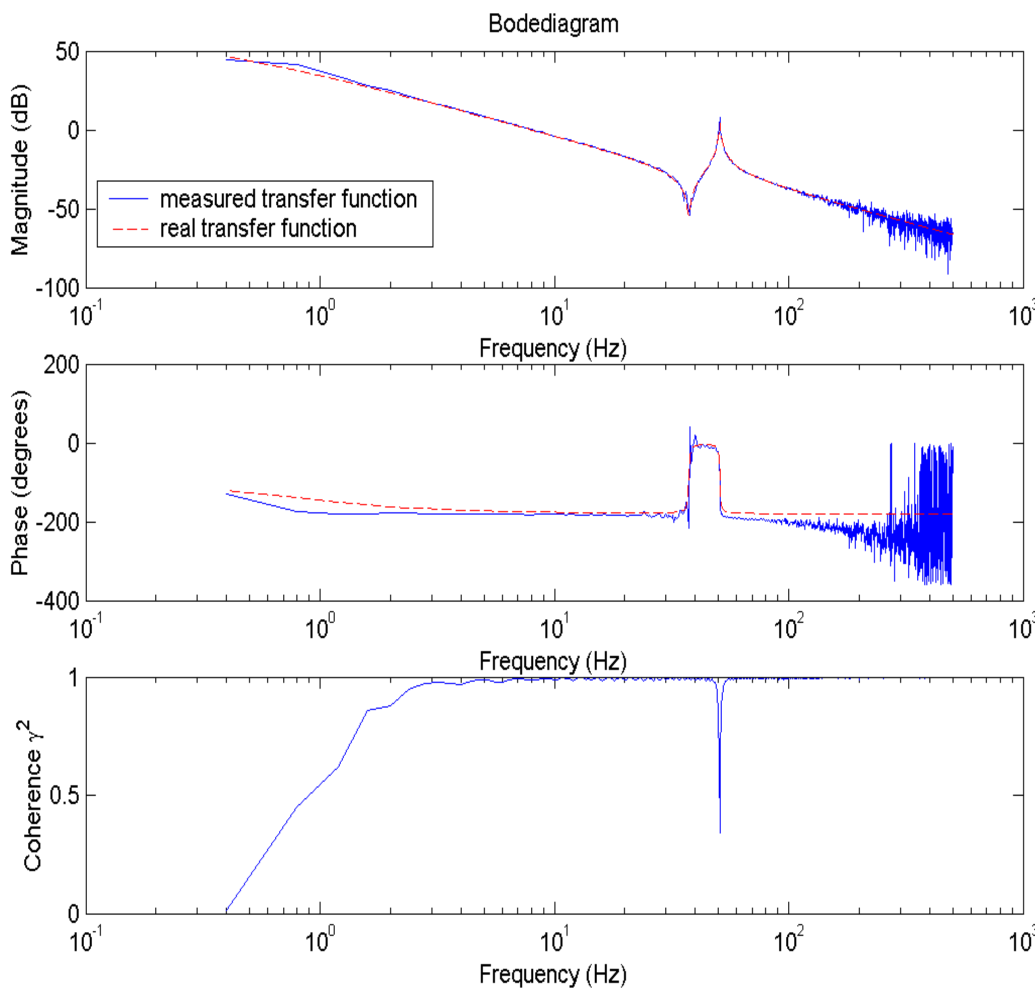

Figure 3.2: Bodediagram and coherence of a sensitivity measurement

This measurement has been taken out with a sampling frequency of 1000 Hz. The total measurement time is 125 s. This has been averaged 50 *times* without overlap which makes 2.5 s per measuring block. The frequency resolution is therefore 0.4 Hz. The solid blue line of the calculated FRF falls right on the dashed red predicted line. This shows that it is a good measurement.

The coherence has the tendency to go to one for higher frequencies. When looking closer, it isn't a smooth, but a wrinkled line and it keeps a steady offset. This is typical for a sensitivity measurement. When the FRF of the system will approach zero for higher frequencies, the sensitivity will approach one. A significant error on the FRF will barely influence the sensitivity, but enough to cause the little offset from one. This is why the coherence always will approach one for small frequency responses. Therefore it needs a critical view.

The dip in the coherence at the resonance frequency also attracts attention. Although I'm not totally sure, my explanation is that, the dip in the coherence is caused by the low sensitivity (-20 dB) at that frequency. When low gains apply, the measurement is sensitive to noise. The power of the noise at point $u(t)$ is relatively large compared with the power caused by signal $w(t)$ at point $u(t)$ when the sensitivity is low.

A trivial remark is to use the right controller by reconstructing the system according to equation (2.25). Still it is easy to make a mistake doing this and it can be useful to measure the controller, just to be sure.

3.4 Effect of noise using the direct method

When extraneous noise applies on the system using the direct method, according formula (2.20) a biased estimate will be measured. When there is mainly extraneous noise acting on the system, even $-1/C$ will be measured.

$$\frac{S_{uy}}{S_{uu}} = \frac{C^*CHS_{rr} + HS_{ww} - C^*H^*HS_{nn} - C^*S_{mm}}{C^*CS_{rr} + S_{ww} + C^*CH^*HS_{nn} + C^*CS_{mm}} \tag{3.1}$$

This phenomenon is simulated by 'nagging' the system during the test. The rotating masses are touched and pushed by hand at approximately a random character. In this way large disturbances are created. For the sample time, number of samples and operations the same values as in section 3.3 are used. When not mentioned, these values are used from now on.

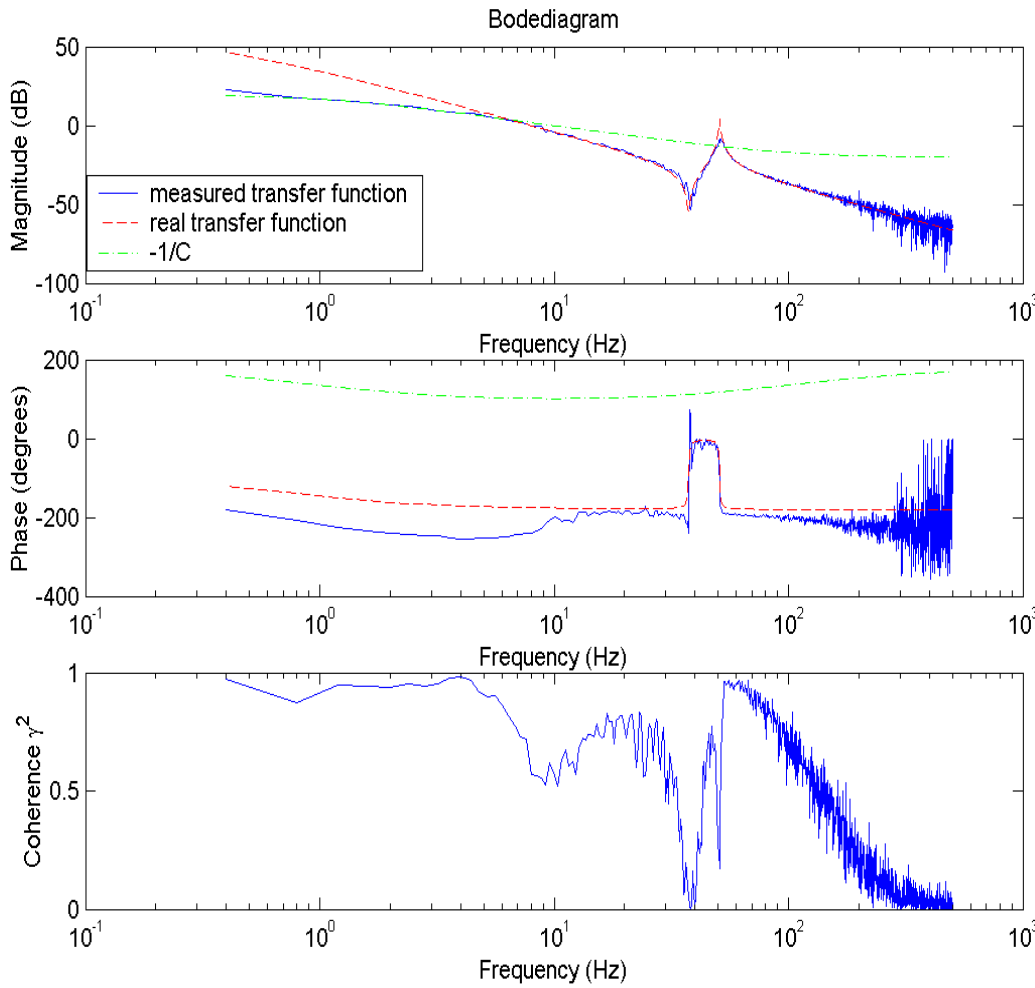

Figure 3.3: Bodediagram and coherence of a 'nagged' system, using direct method

When looking at the results, below a certain frequency the line of $-1/C$ is followed. The main reason for this is the multiplication of the noise $N(f)$ times $|H|^2$. In the lower frequency range $|H|$ will have a large value. An other reason can be found in the hand-applied noise which mainly contains lower frequency. Also notable is that in this range the coherence is closely to one as expected (2.21).

In the regions where the noise doesn't effect the measurement as much, still a good estimate is vested. A main problem when measuring a system what no model exists of, is the coherence which gives no clarity about the correctness of the FRF as described in subsection 2.2.2.

3.5 Leakage and Aliasing

The fourier transform is used for the determination of the FRF. Therefore, the same phenomena which occur during the making of a fourier transform occur at the determined FRF.

For the lower frequencies, long data samples are required. When they are not long enough and have a bad periodic behavior, bad estimates for the lower frequencies are made. Sharp peaks may cause leakage and there is the chance of aliasing. Peaks caused by aliasing can be detected by changing the sampling frequency. Peaks which will 'make a walk' are mirrored back at a half times the sampling frequency from a higher frequency by aliasing.

3.6 Effects of correlated noise

In chapter (2.2) is assumed there exists no correlated noise between different sources, i.e. $S_{rw} = S_{rn} = S_{rm} = S_{wn} = ... = 0$. There are several cases this correlation can occur. The relevant cross-terms of chapter (2.2) have to be taken along. In this section, for one specific case this analysis is detailed.

The jogmode has been set to a speed of 10 rotations/second. Due to the unroundness of the mass, the oscillation of the telescopic bar, commutation of the motor, etc. this will take care of a continuous disturbance $n(t)$ with a period 0.1 s. As signal $w(t)$ for a chirp[1] is chosen which walks through the whole frequency range in five seconds. Now the signals $w(t)$ and $n(t)$ are no longer uncorrelated. When the therm S_{wn} is taken along, instead of equation(2.23), for the sensitivity follows:

$$\begin{aligned}\hat{S} &= \frac{S_{wu}}{S_{ww}} \\ &= S - SCH\frac{S_{wn}}{S_{ww}} \\ &= \frac{1}{1+CH} - \frac{CH}{1+CH}\frac{S_{wn}}{S_{ww}} \\ &= \frac{1-CH\frac{S_{wn}}{S_{ww}}}{1+CH}\end{aligned} \quad (3.2)$$

[1]A chirp is a sinus like signal which frequency changes in time

And the estimated FRF becomes:

$$\begin{aligned}
\hat{H} &= \frac{1-\hat{S}}{C\hat{S}} \\
&= \frac{1}{C\hat{S}} - \frac{1}{C} \\
&= \frac{1+CH}{C(1-CH\frac{S_{wn}}{S_{ww}})} - \frac{1-CH\frac{S_{wn}}{S_{ww}}}{C(1-CH\frac{S_{wn}}{S_{ww}})} \\
&= \frac{CH(1+\frac{S_{wn}}{S_{ww}})}{C(1-CH\frac{S_{wn}}{S_{ww}})} \\
&= H\frac{1+\frac{S_{wn}}{S_{ww}}}{1-\frac{S_{wn}}{S_{ww}}} \\
&= H\frac{S_{ww}+S_{wn}}{S_{ww}-S_{wn}}
\end{aligned} \tag{3.3}$$

When there exists a correlation between the signal $w(t)$ and $n(t)$, a distorted estimate of the FRF is obtained. According to the above analysis, an increased estimate of the FRF is obtained.

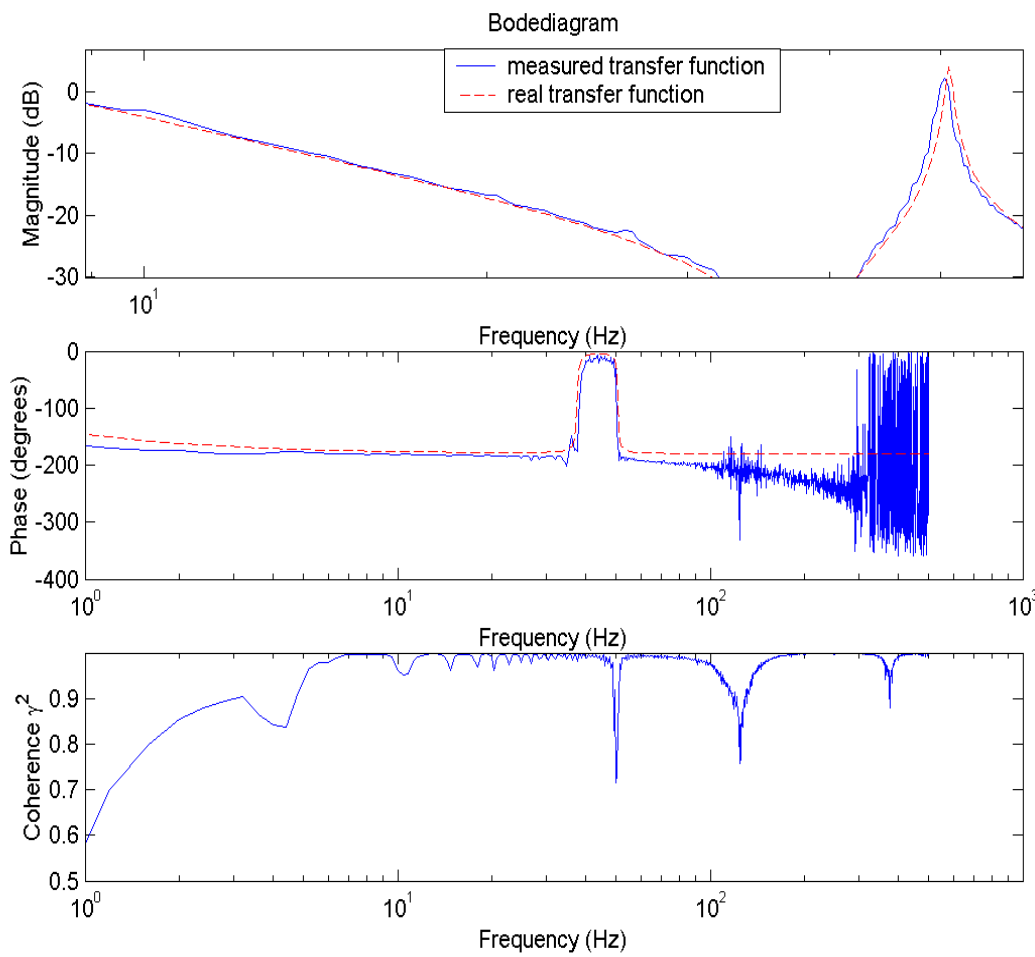

Figure 3.4: Bodediagram and coherence of the system with correlated noise using sensitivity

When we now look at the sensitivity we see a very little wrinkle in the bodediagram an a little dip in the coherence at the frequency of 10 Hz. It could be very small because the correlated disturbance is also very small. In other situations where stronger correlations between different signals exists, stronger distortions will be seen. Using signals that are sensitive for correlating with other signals, like a chirp, a risk is taken. In this example, at other frequencies than 10 Hz, strange phenomena which I can't explain occur.

3.7 Output noise due to AD conversion

In order to get a feeling of the noise at the output due tot analog-digital conversion, the next consideration is proceeded.

Encoder quantization:
2000 pulses/rotation
313.3 pulses/radian
maximum amplitude of noise: $\frac{1}{626.6} rad$

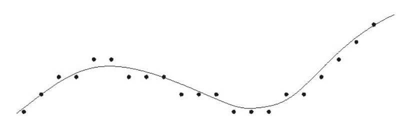

Figure 3.5: quantization

The input of the system has a maximum amplitude of 2.5 Volts. At the output, both signals will have a equivalent amplitude when the absolute value of the FRF is:

$$|H(f)| = \frac{1}{2.5 \cdot 636.3} = -64dB \tag{3.4}$$

When the signal to noise ratio will become too low, bad results will be produced. The variance op the measured FRF will become too large. When the ratio drops below one, instinctive can be said the results will become bad. At least, a lot of averaging steps have to be taken to get reasonable results. When signal to noise ratio become worse, an exponentially increasing time samples have to be averaged to get the same result. In this case, an other effect plays a role. The noise will be no longer uncorrelated with one of the signals from the system. In fact, the encoder quantization depends direct from the output. Imagine the output is rotating so little, the encoder will stay on the same value. With this simple analysis, a sort of boundary is found. For higher frequencies and lower responses, no informative results will be produced.

3.8 Phase delay

In the executed experiments a phase delay has been found. Probably this is caused by the time delays in the signal processing. To become more confident of this assumption, the shape of the delay is checked by a calculation.

The signal processing time delay is assumed constant and called Δt.

$$Phase\ Delay \;=\; 360\Delta t f \;\Rightarrow\; Lineair\ relation \tag{3.5}$$

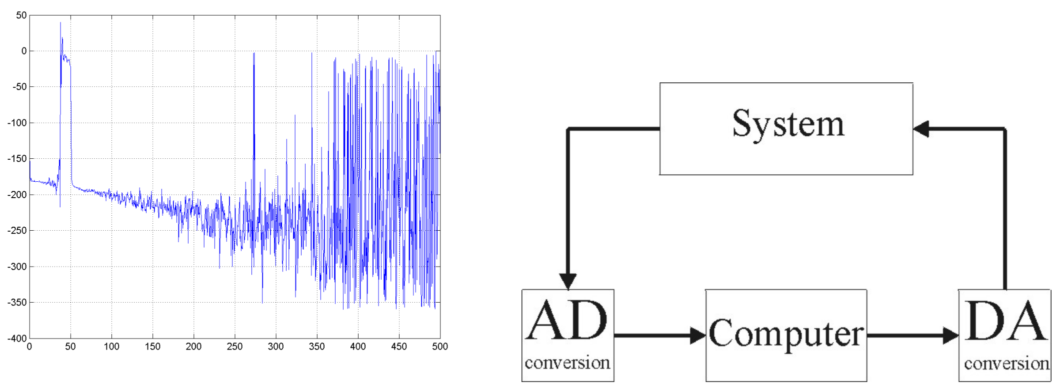

Figure 3.6: Phase delay and proces scheme

At higher frequencies the phase seems to be less accurate. Therefore the phase at lower frequencies is extrapolated. This result in a phase of -300 degrees at 500 Hz. Now the time delay is: $\Delta t = (300 - 180)/(360 * 500) = 0.67 * 10^{-3}\ s$.

The data passes three steps. First, a measurement is token. This can directly be done and causes no time delay. Now the computer has to make its calculations which causes a time delay Δt_1. The calculated values will now pass the DA-conversion with zero order hold operation. This causes a time delay Δt_2.

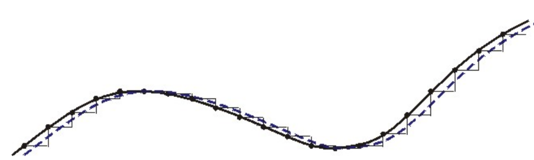

Figure 3.7: Time delay due to DA-conversion

Time delay Δt_1 lies between 0 s and T_{sample} depending on the processor speed. Larger time intervals are not possible, because the computer is then already calculating the next value. $\Delta t_2 = 1/2\ T_{sample}$. For the total time delay counts: $1/2\ T_{sample} < \Delta t < 1\ 1/2\ T_{sample}$. This corresponds with the found value for Δt.

3.9 Detrending

Detrending of signals must be done with care. In normal cases is it even better not to detrend a signal at all. A signal with a steady offset (DC-term) gives no problem for the estimation of any FRF. It only takes care for a large peak at 0 Hz which normally is not even taken along into consideration. Every detrending step is a loss of information. Nevertheless it is still recommendable to detrend a specific case of signals. For example when using the direct method when the system is in jogmode.

According to the lineair superimposition, the following can be done:

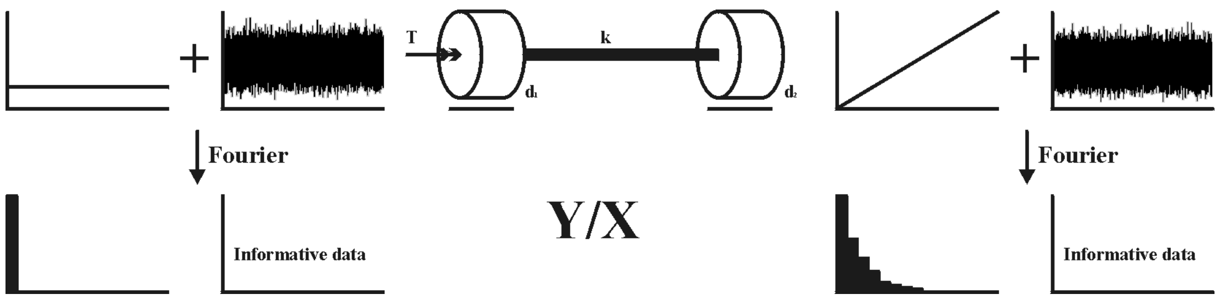

Figure 3.8: Superimposition of signals

When the system is in jogmode, the controller takes care of a constant term which compensates the viscous friction, superpositioned are the other controlleractions. The output of the system describes a ramp with the superimposed informative signal. The continuous term and the ramp contain no information about the system and don't contribute to the determination of the FRF. The continuous term doesn't disturb the fourier transform, but the ramp does. It produces a lot of noise, which makes the variance of the estimated FRF larger and the accuracy worse. By taking longer time samples, the errors due to the jogmode will be averaged out and the accuracy will still be all right. Using the direct method it takes care for a biased estimate.

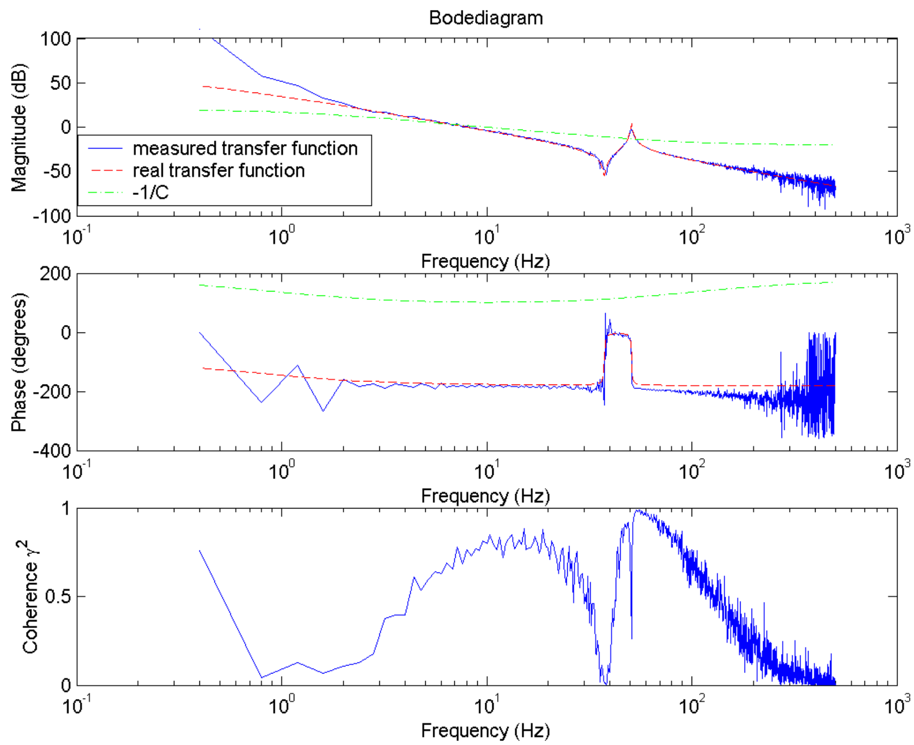

Figure 3.9: Bodediagram and coherence of the system without detrending using the direct method

The FRF of the system using the direct method in jogmode shows big distortions. Especially in the lower frequency regions as expected.

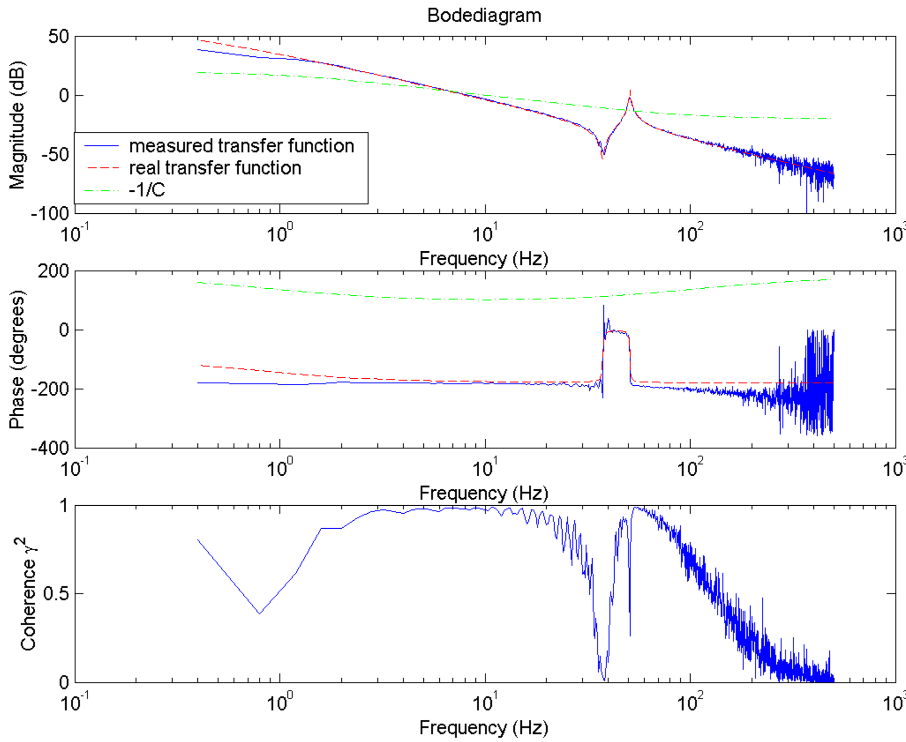

Figure 3.10: Bodediagram and coherence of the system with detrending using the direct method

3.10 Remaining remarks

The theorem of determining the FRF is based on lineair techniques. Be always sure you are working with a lineair system or in a linear region. Watch out for coulomb friction and saturations for protecting the equipment. Therefore it is recommended to always look at the signals in the time domain. Strange phenomena often can be seen easily by looking at the signals. Also problems with detrending can be seen in a glance.

The coherence is a powerful tool in discovering all kinds of problems during an experiment. Nonlinearities, noise and leakage are all found back in the coherence. It pays for itself to try the one and other to see how the coherence reacts.

The most data contains a lot of information. By looking critical to surprising data, often a lot of problems can be solved.

Chapter 4

Conclusion

A frequency response measurement in closed loop is a good method to determine the frequency response function. When the controller is know, the best method to determine the frequency response function is the sensitivity (u/w). It is always useful to look at the coherence, it contains information about the system and the quality of the measurement.

If the controller isn't known, dividing the proces sensitivity by the sensitivity is a good option (y/w*w/u). The direct method also can be used (y/u). There should be taken care, in a noisy situation -1/C can be measured using the direct method.

During a measurement, it is recommended to show a critical look at the results. The assumption of linearity should be satisfied. Signals must be suitable for fourier analyses. Correlations between the input and sources of noise are undesired and take care for distortions. Estimation of the limits of the system give a feeling for which domain a FRF can be estimated.

Appendix A

Closed loop equations in time domain

A.1 S_{uy}

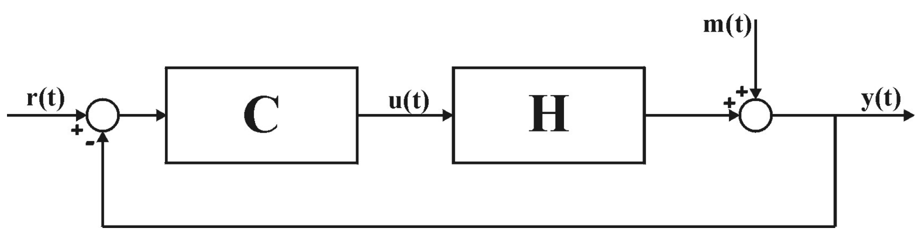

Figure A.1: Simplified scheme of the closed loop system

u(t) can be written as:

$u(t) = c(t) \otimes \Big[r(t) - m(t) - h(t) \otimes u(t)\Big]$

$= \int\limits_0^{+\infty} c(\eta_1)\Big[r(t-\eta_1) - m(t-\eta_1) - \int\limits_0^{+\infty} h(\xi_1)u(t-\eta_1-\xi_1)d\xi_1\Big]d\eta_1$

y(t) can be written as:

$y(t) = h(t) \otimes \Big[c(t) \otimes \Big(r(t) - y(t)\Big)\Big] + m(t)$

$= \int\limits_0^{+\infty}\int\limits_0^{+\infty} c(\eta_2)h(\xi_2)\Big[r(t-\eta_2-\xi_2) - y(t-\eta_2-\xi_2)\Big]d\xi_2 d\eta_2 + m(t)$

The time-shifted-product between the two is:

$$u(t)y(t+\tau) = \Big(\int_0^{+\infty} c(\eta_1)\Big[r(t-\eta_1) - m(t-\eta_1)\Big]d\eta_1 - \int_0^{+\infty}\int_0^{+\infty} c(\eta_1)h(\xi_1)u(t-\eta_1-\xi_1)d\xi_1 d\eta_1\Big)$$
$$\bullet \Big(\int_0^{+\infty}\int_0^{+\infty} c(\eta_2)h(\xi_2)\Big[r(t-\eta_2-\xi_2+\tau) - y(t-\eta_2-\xi_2+\tau)\Big]d\xi_2 d\eta_2 + m(t+\tau)\Big)$$
$$= \int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)h(\xi_2)\Big[r(t-\eta_1)-m(t-\eta_1)\Big]\Big[r(t-\eta_2-\xi_2+\tau)-y(t-\eta_2-\xi_2+\tau)\Big]d\eta_1 d\eta_2 d\xi_2$$
$$+ \int_0^{+\infty} c(\eta_1)\Big[r(t-\eta_1) - m(t-\eta_1)\Big]m(t+\tau)d\eta_1$$
$$- \int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)h(\xi 1)h(\xi_2)u(t-\eta_1-\xi_1)\Big[r(t-\eta_2-\xi_2+\tau)-y(t-\eta_2-\xi_2+\tau)\Big]d\eta_1 d\eta_2 d\xi_1 d\xi_2$$
$$- \int_0^{+\infty}\int_0^{+\infty} c(\eta_1)h(\xi_1)u(t-\eta_1-\xi_1)m(t+\tau)d\eta_1 d\xi_1$$

Cross-correlation between x(t) and y(t) is defined as the expectation of the time-shifted-product between them.

$$R_{xy}(\tau) = E[x(t)y(t+\tau)] = lim_{T\to\infty}\frac{1}{T}\int_0^T x(t)y(t+\tau)dt \tag{A.1}$$

$$\begin{aligned} R_{uy}(\tau) = & \int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)h(\xi_2)\Big(R_{rr}(\eta_1-\eta_2-\xi_2+\tau) - R_{ry}(\eta_1-\eta_2-\xi_2+\tau) \\ & - R_{mr}(\eta_1-\eta_2-\xi_2+\tau) + R_{my}(\eta_1-\eta_2-\xi_2+\tau)\Big)d\eta_1 d\eta_2 d\xi_2 \end{aligned}$$
$$+ \int_0^{+\infty} c(\eta_1)\Big(R_{mr}(\eta_1+\tau) - R_{mm}(\eta_1+\tau)\Big)d\eta_1$$
$$\begin{aligned} - & \int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)h(\xi 1)h(\xi_2)\Big(R_{ur}(\eta_1-\eta_2+\xi_1-\xi_2+\tau) \\ & - R_{uy}(\eta_1-\eta_2+\xi_1-\xi_2+\tau)\Big)d\eta_1 d\eta_2 d\xi_1 d\xi_2 \end{aligned}$$
$$- \int_0^{+\infty}\int_0^{+\infty} c(\eta_1)h(\xi_1)R_{um}(\eta_1+\xi_1+\tau)d\eta_1 d\xi_1$$

Taking the fourier transform using the following transformations gives:

$$\begin{aligned} e^{-2\pi jf\tau} &= e^{+2\pi jf\eta_1}e^{-2\pi jf\eta_2}e^{-2\pi jf\xi_2}e^{-2\pi jf(\eta 1-\eta_2-\xi_2+\tau)} \\ e^{-2\pi jf\tau} &= e^{+2\pi jf\eta_1}e^{-2\pi jf(\eta_1+\tau)} \\ e^{-2\pi jf\tau} &= e^{+2\pi jf\eta_1}e^{-2\pi jf\eta_2}e^{+2\pi jf\xi_1}e^{-2\pi jf\xi_2}e^{-2\pi jf(\eta 1-\eta_2+\xi_1-\xi_2+\tau)} \\ e^{-2\pi jf\tau} &= e^{2\pi jf\eta_1}e^{2\pi jf\xi_1}e^{-2\pi jf(\eta_1+\xi_1+\tau)} \end{aligned}$$

$$
\begin{aligned}
&t = \eta_1 - \eta_2 - \xi_2 + \tau && dt = d\tau \\
&t = \eta_1 + \tau && dt = d\tau \\
&t = \eta_1 - \eta_2 + \xi_1 - \xi_2 + \tau && dt = d\tau \\
&t = \eta_1 + \xi_1 + \tau && dt = d\tau
\end{aligned}
$$

$$
\begin{aligned}
\int_{-\infty}^{+\infty} R_{uy} e^{-2\pi jf\tau} d\tau = & \int_{0}^{+\infty} c(\eta_1) e^{+2\pi jf\eta_1} d\eta_1 \int_{0}^{+\infty} c(\eta_2) e^{-2\pi jf\eta_2} d\eta_2 \int_{0}^{+\infty} h(\xi_2) e^{-2\pi jf\xi_2} d\xi_2 \\
& \cdot \int_{-\infty}^{+\infty} \Big(R_{rr}(t) - R_{ry}(t) - R_{mr}(t) + R_{my}(t) \Big) e^{-2\pi jft} dt \\
+ & \int_{0}^{+\infty} c(\eta_1) e^{+2\pi jf\eta_1} d\eta_1 \int_{-\infty}^{+\infty} \Big(R_{mr}(t) - R_{mm}(t) \Big) e^{-2\pi jft} dt \\
- & \int_{0}^{+\infty} c(\eta_1) e^{+2\pi jf\eta_1} d\eta_1 \int_{0}^{+\infty} c(\eta_2) e^{-2\pi jf\eta_2} \int_{0}^{+\infty} h(\xi 1) e^{+2\pi jf\xi_1} \int_{0}^{+\infty} h(\xi_2) e^{-2\pi jf\xi_2} \\
& \cdot \int_{-\infty}^{+\infty} \Big(R_{ur}(t) - R_{uy} \Big)(t) e^{-2\pi jft} dt \\
- & \int_{0}^{+\infty} c(\eta_1) e^{2\pi jf\eta_1} d\eta_1 \int_{0}^{+\infty} h(\xi_1) e^{2\pi jf\xi_1} d\xi_1 \int_{-\infty}^{+\infty} R_{um}(t) e^{-2\pi jft} dt
\end{aligned}
$$

Cross-spectral density function is defined as the fourier transform of the cross-correlation.

$$
S_{xy}(f) = \int_{-\infty}^{+\infty} R_{xy}(\tau) e^{-2\pi jf\tau} d\tau \tag{A.2}
$$

$$
X(f) = \int_{-\infty}^{+\infty} x(t) e^{-2\pi jft} dt \tag{A.3}
$$

$$
S_{xy}(-f) = S^*_{xy}(f) = S_{yx}(f) \tag{A.4}
$$

The assumption that r(t) and m(t) are uncorrelated implies that $R_{mr} = R_{rm} = 0$

$$S_{uy}(f) = C^*(f)C(f)H(f)[S_{rr}(f) - S_{ry}(f) + S_{my}(f)]$$
$$- C^*(f)S_{mm}(f)$$
$$+ C^*(f)C(f)H^*(f)H(f)[S_{uy}(f) - S_{ur}(f)]$$
$$- C^*(f)H^*(f)S_{um}(f)$$

In the same way can be found:
$S_{my} = CH[S_{mr} - S_{my}] + S_{mm}$
$S_{ry} = CH[S_{rr} - S_{ry}] + S_{rm}$
$S_{ur} = C^*[S_{rr} - S_{mr}] - C^*H^*S_{ur}$
$S_{um} = C^*[S_{rm} - S_{mm}] - C^*H^*S_{um}$

Taking into account that $S_{mr} = S_{rm} = 0$ and where $S = \frac{1}{1+CH}$

$S_{ry} = CHSS_{rr}$
$S_{my} = SS_{mm}$
$S_{ur} = C^*S^*S_{rr}$
$S_{um} = -C^*S^*S_{mm}$

$$S_{uy}(1-C^*CH^*H) = [C^*CH-C^*CCHHS-C^*C^*CH^*HS^*]S_{rr}+[-C^*+C^*CHS+C^*C^*H^*S^*]S_{mm}$$

A.2 S_{uu}

The time-shifted-product between u(t) and u(t) is:

$$u(t)u(t+\tau) = \Big(\int_0^{+\infty} c(\eta_1)\Big[r(t-\eta_1) - m(t-\eta_1)\Big]d\eta_1 - \int_0^{+\infty}\int_0^{+\infty} c(\eta_1)h(\xi_1)u(t-\eta_1-\xi_1)d\xi_1 d\eta_1\Big)$$
$$\bullet \Big(\int_0^{+\infty} c(\eta_2)\Big[r(t-\eta_2+\tau) - m(t-\eta_2+\tau)\Big]d\eta_2 - \int_0^{+\infty}\int_0^{+\infty} c(\eta_2)h(\xi_2)u(t-\eta_2-\xi_2+\tau)d\xi_2 d\eta_2\Big)$$

$$R_{uu}(\tau) = \int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)\Big[R_{rr}(\eta_1-\eta_2+\tau) + R_{mm}(\eta_1-\eta_2+\tau)\Big]d\eta_1 d\eta_2$$
$$-\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)h(\xi_2)\Big[R_{ru}(\eta_1-\eta_2-\xi_2+\tau) - R_{mu}(\eta_1-\eta_2-\xi_2+\tau)\Big]d\eta_1 d\eta_2 d\xi_2$$
$$-\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)h(\xi_1)\Big[R_{ur}(\eta_1-\eta_2+\xi_1+\tau) - R_{um}(\eta_1-\eta_2+\xi_1+\tau)\Big]d\eta_1 d\eta_2 d\xi_1$$
$$+\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty}\int_0^{+\infty} c(\eta_1)c(\eta_2)h(\xi_1)h(\xi_2)\Big[R_{uu}(\eta_1-\eta_2+\xi_1-\xi_2+\tau)\Big]d\eta_1 d\eta_2 d\xi_1 d\xi_2$$

In analogy of the calculation of S_{uy}, the fourier transform of the first variable becomes conjugated.

$$S_{uu}(f) = C^*(f)C(f)[S_{rr}(f) + S_{mm}(f)]$$
$$+ C^*(f)C(f)H(f)[S_{mu}(f) - S_{ru}(f)]$$
$$+ C^*(f)C(f)H^*(f)[S_{um}(f) - S_{ur}(f)]$$
$$+ C^*(f)C(f)H^*(f)H(f)[S_{uu}(f)]$$

$$S_{uu}(1-C^*CH^*H) = [C^*C - C^*CCHS - C^*C^*CH^*S^*]S_{rr} + [C^*C - C^*CCHS - C^*C^*CH^*S^*]S_{mm}$$

A.3 Estimated frequency response function

The estimated FRF becomes:

$$\frac{S_{uy}}{S_{uu}} = \frac{[C^*CH - C^*CCHHS - C^*C^*CH^*HS^*]S_{rr} + [-C^* + C^*CHS + C^*C^*H^*S^*]S_{mm}}{[C^*C - C^*CCHS - C^*C^*CH^*S^*]S_{rr} + [C^*C - C^*CCHS - C^*C^*CH^*S^*]S_{mm}}$$

$$\frac{S_{uy}}{S_{uu}} = \frac{C^*CH[1 - CHS - C^*H^*S^*]S_{rr} - C^*[1 - CHS - C^*H^*S^*]S_{mm}}{C^*C[1 - CHS - C^*H^*S^*]S_{rr} + C^*C[1 - CHS - C^*H^*S^*]S_{mm}}$$

$$\frac{S_{uy}}{S_{uu}} = \frac{C^*CHS_{rr} - C^*S_{mm}}{C^*CS_{rr} + C^*CS_{mm}}$$

The above is a very indirect manner for the derivation of the estimated FRF. Nevertheless it gives a good feeling how to deal with correlations and spectra.

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